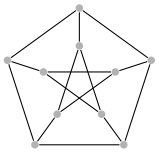
Name: _____

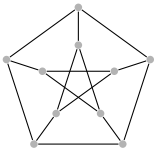
Directions: Show all work.

1. **[3 parts, 2 points each]** Let G be the Petersen graph. Recall that $V(G)$ is the set of 2-element subsets of $\{1, 2, 3, 4, 5\}$ with $uv \in E(G)$ if and only if u and v are disjoint.

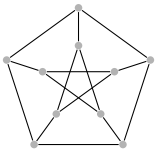


   (a) Is G bipartite? Prove your answer is correct.

   (b) Recall that P_n is the path with n vertices. Let k be the maximum integer such that P_k is a subgraph of G. Determine k and find a copy of P_k as a subgraph of G. No proof required.



   (c) Let t be the maximum integer such that P_t is an *induced* subgraph of G. Determine t and find a copy of P_t as an *induced* subgraph of G. No proof required.



2. **[2 parts, 2 points each]** Graph Ramsey Problems.

   (a) Prove that $K_5 \not\to (P_5, P_5)$.

   (b) Recall that C_n is the n-vertex cycle. Use the fact that $K_6 \to (C_4, C_4)$ to prove that $K_6 \to (P_5, P_5)$.