Name: _____

Directions: Show all work.

1. Let $U = [n]^2 = \{(x, y) \colon x, y \in \{1, \ldots, n\}\}$.

   (a) **[4 points]** Use U to give a combinatorial proof that $n^2 = 2\binom{n}{2} + n$.

   (b) **[3 points]** Use part (a) and an identity from HW12 to give a formula for $\sum_{k=1}^{n} k^2$.

2. **[3 points]** Let n be a positive integer. Let A be the set of all triples $\{x, y, z\} \in \binom{[n]}{3}$ such that $x < y < z$ and $z - y = y - x$. Let B be the set of all pairs $\{a, b\} \in \binom{[n]}{2}$ such that $a < b$ and $b - a$ is even. Give a bijection to show that $|A| = |B|$.