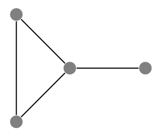
Name: Solutions

Directions: Show all work.

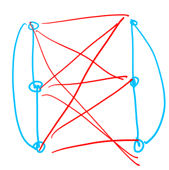
1. **[2 parts, 5 points each]** Let G be the 4-vertex graph consisting of a vertex u of degree 3 plus one additional edge joining two neighbors of u; this graph is called the paw.



G

(a) Prove that $r(G, G) > 6$.

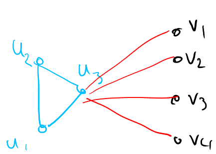
We give a {blue, red}-edge-coloring of K_6 that avoids a monochromatic copy of G:



The blue components each have 3 vertices, so there is no blue copy of G. Also, the red component is $K_{3,3}$, and hence the red subgraph is bipartite and has no copy of G (as G has a triangle).

(b) Prove that $r(G.G) \le 7$.

Let H be a {blue, red}-edge-coloring of K_7. Since $r(3,3) = 6 < 7$, it follows that H has a monochromatic triangle $u_1 u_2 u_3$. Without loss of generality, we may assume $u_1 u_2 u_3$ is blue. Let $v_1, \dots, v_4$ be the other vertices in H. If there is a blue edge from u_3 to a vertex in $\{v_1, v_2, v_3, v_4\}$, then H has a blue copy of G. So suppose every edge from u_3 to vertices in $\{v_1, \dots, v_4\}$ is red, as shown.



If $v_i v_j$ is red, then u completes a red triangle with $v_i v_j$ and any other red neighbor of u completes a red copy of G. Otherwise, every edge in $\{v_1, \dots, v_4\}$ is blue, and so we obtain a blue copy of K_4, which contains a blue copy of G. In all cases, there is a mono. copy of G, implying $K_7 \to (G, G)$ and $r(G, G) \le 7$. ∎