Name: Solutions

Directions: Show all work.

1. **[3 parts, 2 points each]** Let G be the graph whose vertices are the 2-element subsets of $\{1,\ldots,10\}$ such that u and v are adjacent if and only if $|u \cap v| = 1$.

   (a) How many vertices does G have? Give a simplified numerical answer.

$$|V(G)| = \binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{10!}{2!\cdot 8!} = \frac{\cancel{10}^{5}\cdot 9}{\cancel{2}\cdot 1} = 9\cdot 5 = \boxed{45}$$

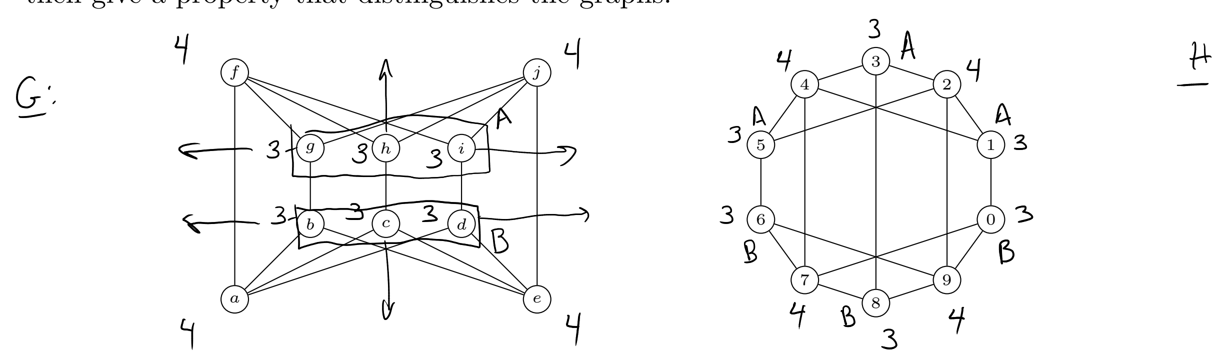
   (b) How many edges does G have? Give a simplified numerical answer.

- To be adjacent to $\{a,b\}$ we need to form a 2-set with one elt in $\{a,b\}$ (2 options) plus one elt from $\{1,\ldots,10\} - \{a,b\}$ (8 options). So degree is $2\cdot 8$ or 16.
- $|E(G)| = \frac{1}{2}\sum_v d(v) = \frac{1}{2}\sum_v 16 = \frac{1}{2}\cdot(45)(16) = 45\cdot 8 = 320+40 = \boxed{360}$.

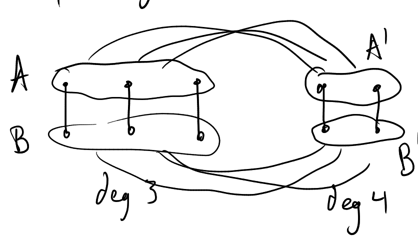
   (c) Is G bipartite? Explain why or why not.

$\boxed{\text{No}}$. Since G contains odd cycles, like the triangle $\{1,2\}$ $\{1,3\}$ $\{1,4\}$, G is not bipartite.

2. **[4 points]** Are the following graphs isomorphic? If so, then give an isomorphism. If not, then give a property that distinguishes the graphs.



Note: An isomorphism must preserve the degrees of vertices, so $\{a,f,e,j\}$ must map in some way to $\{2,4,7,9\}$. Vertices of degree 3 form a matching, and the vertices of degree 4 also form a matching.



Between A and A', we have a complete bipartite graph. Also between B and B'.

So these graphs are $\boxed{\text{isomorphic}}$; many isomorphisms are possible, for example:

V(G)	a	b	c	d	e	f	g	h	i	j
V(H)	7	6	8	0	9	4	5	3	1	2

V(H)	0	1	2	3	4	5	6	7	8	9
V(G)	d	i	j	h	f	g	b	a	c	e