Name: \_

**Directions:** Show all work.

1. Consider the  $(n \times n)$ -grid, with set of cells  $\{(x, y): 0 \le x \le n - 1 \text{ and } 0 \le y \le n - 1\}$ . Two cells (x, y) and (x', y') share a diagonal if y + x = y' + x' or y - x = y' - x'. Note that when y + x = y' + x', both cells are on the same diagonal with slope -1 and when y - x = y' - x', both cells are on the same diagonal with slope 1. For example, the cells that share a diagonal with (3, 5) in the  $(8 \times 8)$ -grid are shaded below left.



- (a) [3 points] For  $n \in \{2, 3, 4\}$ , give a maximum-sized set of cells with no two cells sharing a diagonal in the grids above right. (No need to prove your answer is correct.)
- (b) [2 points] For  $n \ge 2$ , make a conjecture for the maximum size of a set of cells in the  $(n \times n)$ -grid with no two cells sharing a diagonal.
- (c) [5 points] Prove that your conjecture is correct.