Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. [SS 1.3.1] Let $a_0 = 0$ and $a_n = 3a_{n-1} + 2$ for $n \geq 1$.
   (a) Find the first few values of the sequence a_n and use this to guess a general formula.
   (b) Use induction to prove that your general formula from part (a) is correct.
2. Let a_n be the number of lists of length n with entries in $\{0, 1, 2\}$ without two consecutive zeros. Note that $a_0 = 1$ (since the empty list does not have consecutive zeros), $a_1 = 3$, and $a_2 = 8$ (since all 9 lists of length 2 are counted except 00).
   (a) Find a second order homogeneous recurrence relation for a_n. In other words, find constants s and t such that $a_n = sa_{n-1} + ta_{n-2}$ for $n \geq 2$. Remember to include base cases and argue that your recurrence relation is correct.
   (b) Use part (a) to explicitly compute a_n for $0 \leq n \leq 6$.
   (c) Use the characteristic equation method to solve your recurrence in part (a) to find an explicit formula for a_n.
3. Prove that if it is possible to tile an $m \times n$ grid with 4×1 rectangular tiles, then at least one of the side lengths is divisible by 4. (Hint: find a way to color the grid with 4 colors so that each tile covers one cell of each color.)