Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

- 1. [10.1.13] Prove that d_1, \ldots, d_p is graphic if and only if $p 1 d_1, \ldots, p 1 d_p$ is graphic.
- 2. [10.2.1] Let x and y be vertices of a graph.
 - (a) Suppose that there is a closed walk containing both x and y. Must there be a closed trail containing both x and y?
 - (b) Suppose that there is a closed trail containing both x and y. Must there be a cycle containing both x and y?
- 3. [10.2.8] Let G be a forest consisting of t trees, and let n = |V(G)|. How many edges does G have? Prove your answer is correct.
- 4. [10.2.17] Let $n \ge 2$ and let $d_1 \ge d_2 \ge \cdots \ge d_n$ be a sequence of positive integers. Prove that this sequence is the degree sequence of a tree if and only if $\sum_{i=1}^{n} d_i = 2n 2$.