

**Directions:** Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. [5.2.4] Find a simple expression for  $\sum_{k=0}^n (2k+1) \binom{n}{k}$ .
2. Let  $n \geq 1$ , let  $O_n$  be the set of odd numbers in  $\{0, 1, \dots, n\}$ , and let  $E_n$  be the set of even numbers in  $\{0, 1, \dots, n\}$ . Give a combinatorial proof that  $\sum_{k \in E_n} \binom{n}{k} = \sum_{k \in O_n} \binom{n}{k}$ .
3. [5.2.9] Let  $n \geq 1$ , let  $O_n$  be the set of odd numbers in  $\{0, 1, \dots, n\}$ , and let  $E_n$  be the set of even numbers in  $\{0, 1, \dots, n\}$ . Let  $a_n = \sum_{k \in E_n} k \binom{n}{k}$  and  $b_n = \sum_{k \in O_n} k \binom{n}{k}$ .
  - (a) Use the binomial theorem to find expressions for  $a_n + b_n$  and  $a_n - b_n$ .
  - (b) Find formulas for  $a_n$  and  $b_n$ .
4. [8.1.3] How many permutations of the letters in SCRIPPS have no two consecutive letters the same?
5. How many permutations of the letters in AABBBCC...ZZ have no two consecutive letters the same? Find a summation formula.
6. [10.1.2] Is  $7, 7, 6, 5, 4, 4, 4, 3, 2$  a graphic sequence?