Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[9 parts, 2 points each]** Decide whether the following are true or false. Indicate your answer by writing the entire word. No justification required.

$$A = \{(1,3), \{1,3\}\} \qquad B = \{1,3\} \qquad C = \{\{3,1\}\} \qquad D = \{(3,1)\} \qquad E = \{\{\{1,3\}\}\}$$

(a) $3 \in B$

TRUE

(b) $3 \in C$

FALSE. C contains one set

(c) $B \in A$

TRUE.

(d) $B \in C$

TRUE

(e) $3 \subseteq B$

FALSE. 3 is not a set

(f) $B \subseteq C$

FALSE. $1 \in B$ but $1 \notin C$

(g) $C \subseteq A$

TRUE The only elt in C, namely $\{1,3\}$, is also in A.

(h) $A \subseteq D$

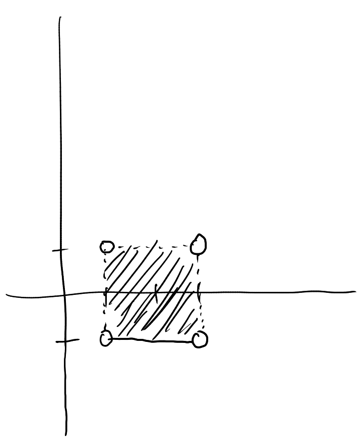
FALSE

(i) $C \subseteq E$

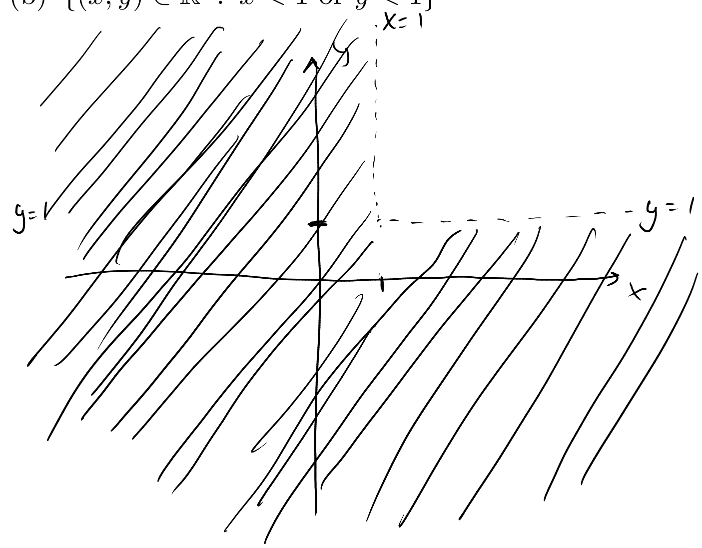
FALSE $\{1,3\} \in C$ but $\{1,3\} \notin E$ since $\{1,3\} \neq \{\{1,3\}\}$

2. **[2 parts, 3 points each]** Sketch the following sets in the plane.

(a) $(1,3) \times [-1,1)$



(b) $\{(x,y) \in \mathbb{R}^2 : x < 1 \text{ or } y < 1\}$



3. **[6 parts, 3 points each]** Express each set by listing the elements between braces.

$A = \{\{\}, 2, \{1\}, \{2, 2\}\}$ $\quad B = \{\{1, 1\}, \{2, 3\}, (2, 3), 2\}$ $\quad C = \{\{2\}, \{3, 2\}, \varnothing\}$ $\quad D = \{\varnothing, \{1, 2\}, \{2, 3\}, (3, 2)\}$

(a) $A \cap B$

$\boxed{\{2, \{1\}\}}$

(b) $B \cap C$

$\boxed{\{\{2,3\}\}}$

(c) $(B \cup C) - A$

$\boxed{\{\{2,3\}, (2,3)\}}$

(d) $(C - A) \times C$

$C - A = \{\{2,3\}\}$

$(C-A) \times C = \boxed{\{(\{2,3\}, \{2\}), (\{2,3\}, \{2,3\}), (\{2,3\}, \varnothing)\}}$

(e) $\mathcal{P}(C \cap D)$

$C \cap D = \{\overbrace{\{2,3\}}^{a}, \overbrace{\varnothing}^{b}\}$

$\mathcal{P}(C \cap D) = \{\varnothing, \{a\}, \{b\}, \{a,b\}\}$

$= \boxed{\{\varnothing, \{\{2,3\}\}, \{\varnothing\}, \{\{2,3\}, \varnothing\}\}}$

(f) $A \cap \mathcal{P}(\mathbb{Z})$

$\mathcal{P}(\mathbb{Z})$ = all subsets of the set of integers

$A \cap \mathcal{P}(\mathbb{Z}) = \boxed{\{\varnothing, \{1\}, \{2\}\}}$

4. **[3 parts, 4 points each]** Give an example of a set with the following properties or explain why no such set exists.

(a) A set $A \subseteq \mathbb{N}$ such that A and $\overline{A}$ are both infinite.

$\boxed{A = \{n \in \mathbb{N} : n \text{ is even}\}}$ Many answers possible.

(b) A set $B \subseteq \mathbb{Z}$ such that every integer in B is positive and every integer in B is negative.

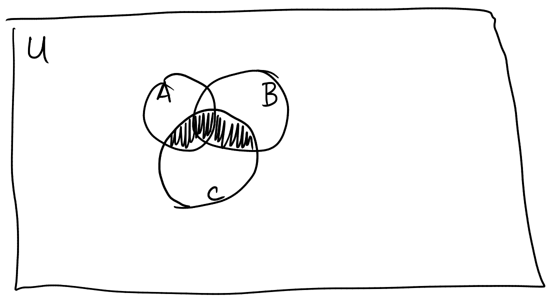
$\boxed{B = \varnothing}$. Only 1 answer possible.

(c) A finite set C such that $\mathcal{P}(C)$ is infinite.

$\boxed{\text{There is no such set.}}$ If $|C| = n$, then $|\mathcal{P}(C)| = 2^n$. So if C is finite, then so is $\mathcal{P}(C)$.

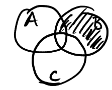
5. **[4 parts, 3 points each]** Give Venn Diagrams for each of the following sets relative to a universe U.

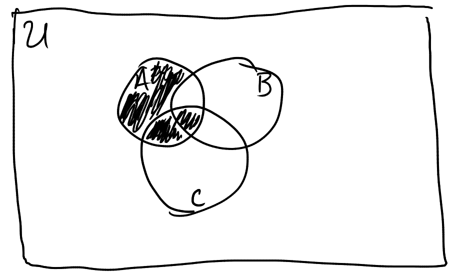
(a) $(A \cup B) \cap C$



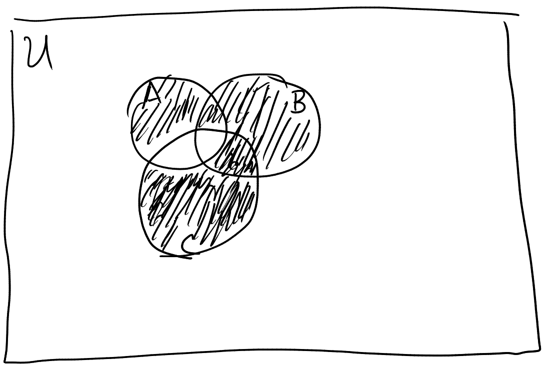
(c) $A - (B - C)$

$B - C$:

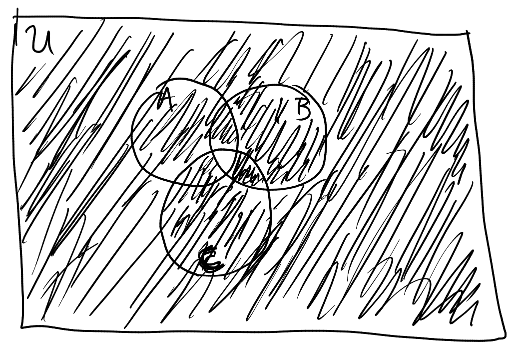




(b) $(A \cup B \cup C) - (A \cap C)$



(d) $\overline{A \cap B} \cup \overline{B \cap C}$



6. **[5 points]** Give two examples of an infinite set A such that $A \in \mathcal{P}(\mathcal{P}(\mathbb{R}))$. $\iff A \subseteq \mathcal{P}(\mathbb{R})$

$\iff$ Every elt in A is a subset of $\mathbb{R}$

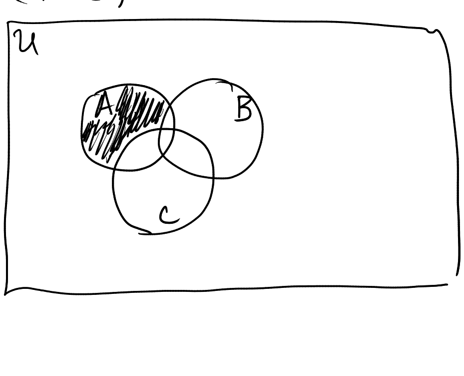
$A = \{ [a,b] : a, b \in \mathbb{R} \text{ and } a \leq b \}$

Many answers possible.

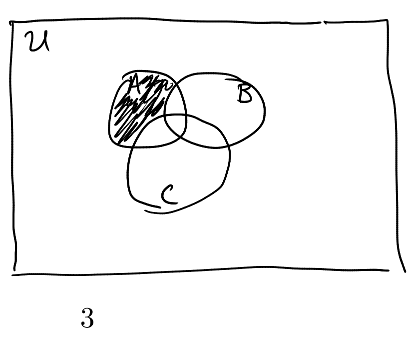
$A = \{ (a,b) : a \,\&\, b \in \mathbb{R} \text{ and } a < b \}$

7. **[5 points]** Use Venn Diagrams to decide if the equation $(A - B) - C = A \cap \overline{B} \cap \overline{C}$ is valid for all sets A, B, and C.

$(A - B) - C$



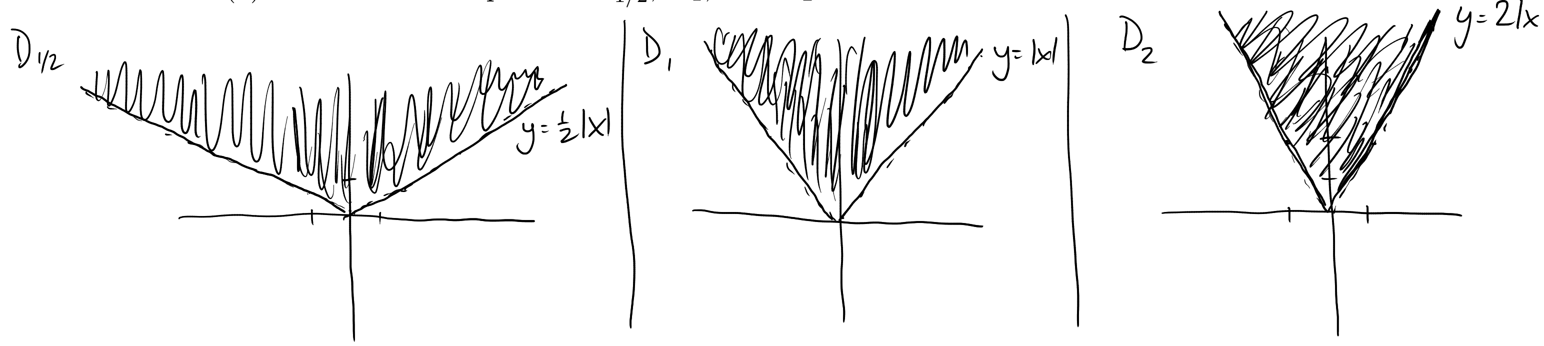
$A \cap \overline{B} \cap \overline{C}$



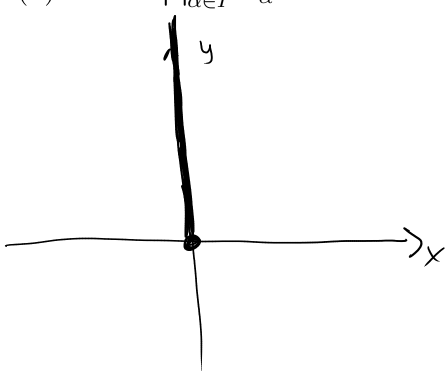
Yes, this equation is valid for all sets A, B, C.

8. **[3 parts, 6 points each]** Let $I = \{\alpha \in \mathbb{R} \colon \alpha > 0\}$ and let $D_\alpha = \{(x, y) \in \mathbb{R}^2 \colon y \geq \alpha|x|\}$. Note that $|x|$ is the absolute value of x, so that $|x| = x$ when $x \geq 0$ and $|x| = -x$ when $x < 0$.

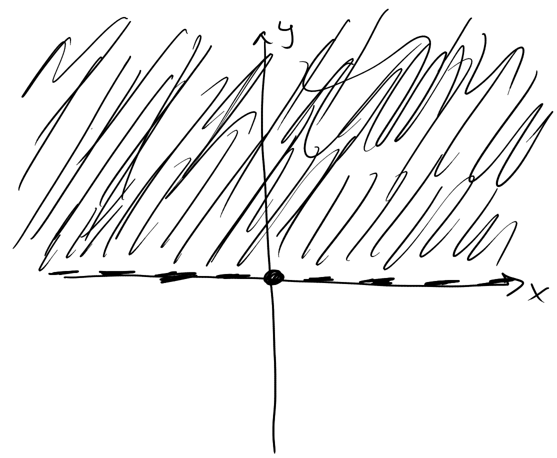
   (a) Sketch the example sets $D_{1/2}$, D_1, and D_2.



   (b) Sketch $\bigcap_{\alpha \in I} D_\alpha$.

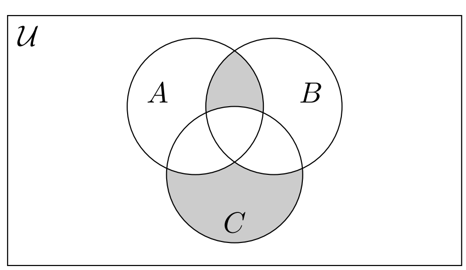


   (c) Sketch $\bigcup_{\alpha \in I} D_\alpha$.



9. **[6 points]** Express the shaded portion of the following Venn diagram as a set by applying elementary set operations to A, B, and C.

Several Answers possible



$\boxed{((C - A) - B) \cup ((A \cap B) - C)}$ or $\boxed{(C - (A \cup B)) \cup ((A \cap B) - C)}$