

Name: \_\_\_\_\_

**Directions:** Solve the following problems. Give supporting work/justification where appropriate.

1. Prove the following using a direct proof.

(a) **[4 points]** Let  $a, b \in \mathbb{Z}$ . We have that  $a + b$  is even if and only if  $a - b$  is even.(b) **[3 points]** Let  $x \in \mathbb{R}$ . If  $x \neq 1$ , then  $x < \frac{x^2+1}{2}$ .

- (c) **[3 points]** Let  $a \in \mathbb{Z}$ . If  $5 \mid 8a$ , then  $5 \mid a$ . (Hint: the equation  $a = 16a - 15a$  may be useful.)