Name: ______________________________

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[9 parts, 2 points each]** Decide whether the following are true or false. Indicate your answer by writing the entire word. No justification required.

$$A = \{\varnothing\} \quad B = \{1,2,3\} \quad C = \{\{1,2\},\{3\}\} \quad D = \{\varnothing,\{1,2,3\}\} \quad E = \{\{2,1\},\{2,3\}\}$$

(a) $3 \in B$

(b) $3 \in C$

(c) $A \in D$

(d) $B \in D$

(e) $2 \subseteq B$

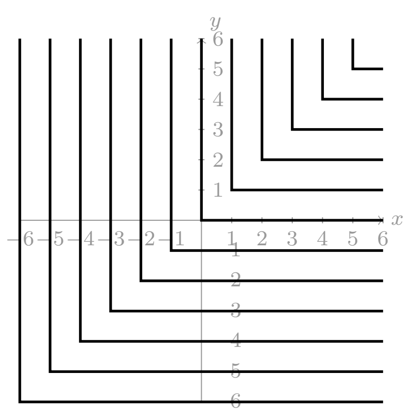
(f) $B \subseteq C$

(g) $B \subseteq D$

(h) $A \subseteq D$

(i) $C \subseteq E$

2. **[6 points]** A subset of $\mathbb{R}^2$ is sketched below; the pattern in the figure below continues throughout the plane. Use set-builder notation to give a simple description of the set.



3. **[6 parts, 3 points each]** Express each set by listing the elements between braces.

$A = \{\{\}, \{1,2\}, \{2,1\}\}$ $B = \{\varnothing, 2, \{1,2\}, (1,2)\}$ $C = \{\varnothing, 1+1, \{2,1\}, (2,1)\}$ $D = \{\{1\}, 1, \{2\}, 2\}$

(a) $A \cap B$

(b) $B \cap C$

(c) $(B \cup C) - A$

(d) $(C - A) \times A$

(e) $\mathcal{P}(B \cap D)$

(f) $(A \cup B \cup C) \cap \mathbb{Z}^2$

4. **[3 parts, 4 points each]** Give an example or explain why no examples exist.

(a) A set A such that $(1,2) \in \mathcal{P}(A)$.

(b) Sets A and B such that $|A \times B| = 3$.

(c) A nonempty set A such that $A \subseteq \mathcal{P}(A)$.

5. **[4 parts, 3 points each]** Give Venn Diagrams for each of the following sets relative to a universe U.

   (a) $(A - B) \cup (B - A)$

   (c) $(C \cup B) \cap A$

   (b) $(A \cup C) \cap (B \cup C)$

   (d) $\overline{A \cup B} \cup (A \cap B \cap C)$

6. **[5 points]** Give two examples of an infinite set A such that $A \in \mathcal{P}(\mathcal{P}(\mathbb{Z}))$.

7. **[5 points]** Use Venn Diagrams to decide if the equation $(A \cap B) - C = (A - C) \cap B$ is valid for all sets A, B, and C.

8. **[3 parts, 6 points each]** Let $D_\alpha = \{(x, y) \in \mathbb{R}^2 \colon (x - \alpha)^2 + y^2 \leq \alpha^2 + 1^2\}$. In English, D_α is the closed disk with center at $(\alpha, 0)$ whose circumference passes through the points $(0, -1)$ and $(0, 1)$. Let $I = \{\alpha \in \mathbb{R} \colon \alpha \geq 0\}$.

   (a) Sketch the example sets D_0, D_1, and D_2.

   (b) Sketch $\bigcap_{\alpha \in I} D_\alpha$.

   (c) Sketch $\bigcup_{\alpha \in I} D_\alpha$.

9. **[6 points]** Briefly describe Russell's paradox and how mathematicians have addressed it.