Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. [6 parts, 1 point each] Define the following statements and open sentences.

P: For each  $z \in \mathbb{R}$ , we have  $z^2 \ge 0$ .

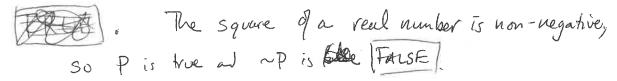
 $Q(x): x \in \mathbb{Z}.$ 

R(x): x is an even integer.

S(A): A is a finite set.

Decide whether the following are true or false; indicate your answer by writing the entire word "true" or the entire word "false". By the faction for partial credit.

(a)**~**P



(b)  $S(\mathcal{P}(\mathbb{R}) \cap \mathbb{R})$ 

Elements of P(R) are sets. Elements of R are real numbers. So  $P(R) \cap R = \emptyset$ .

(c)  $R(3) \Rightarrow S(\mathbb{Z})$ 

TRUE .

Since R(3) is false, the implication is [TRUE].

(d)  $(Q(0)) P \wedge R(6)$ 

GFALSE & TRUE & TRUE -> TRUE -

(e) For all x, we have  $R(x) \Leftrightarrow Q(\frac{x}{2})$ .

TRUE If x is an even integer, then  $\stackrel{\times}{\sim} \pm \mathbb{Z}$ . Also, if  $\stackrel{\times}{\sim} \pm \mathbb{Z}$ , then x is an even integer.

(f)  $(\sim (S(\varnothing) \Rightarrow R(1))) \lor (P \land S(\mathbb{R}^2))$ 

> (~(True >> FALSE)) V (TRUE ^ FALSE) >> (~ FALSE) V (FALSE) >> TRUE V FALSE -> True

- 2. [2 parts, 1 point each] Truth tables and logical equivalence.
  - (a) Write a truth table for  $(P \lor Q) \Rightarrow (P \land Q)$ .

P   Q	PrQ	PAQ	$(P \vee Q) \Rightarrow (P \wedge Q)$
TIT	T	T	T
TF	T	F	F
FIT	T	F	F
FF	F	F	T

(b) Give a simple statement which is logically equivalent to  $(P \lor Q) \Rightarrow (P \land Q)$ .

- 3. [2 parts, 1 point each] Let P, Q, and R be statements. Use the logical operands to express the following statements.
  - (a) P and Q have the same truth value, but R has the opposite truth value.

(b) If at least two of the statements in  $\{P,Q,R\}$  are true, then so is the third.