Name: Solutions

Directions: All questions require explanation in English sentences.

1. **[2 parts, 3 points each]** Find a formula for the following sums.

   (a) $1 + 2 + 3 + \cdots + n$

Let $S = 1 + 2 + 3 + \cdots + n$. We have that

$S = n + (n-1) + (n-2) + \cdots + 1$. Adding these two equations

gives

$$2S = \underbrace{(n+1) + (n+1) + (n+1) + \cdots + (n+1)}_{n \text{ terms}}$$

Therefore $2S = n(n+1)$ and so $S = \boxed{\frac{n(n+1)}{2}}$.

   (b) $1 + 2 + 3 + \cdots + 5n$

Replacing n by $5n$ in the formula for part (a),

we have that

$$1 + 2 + \cdots + 5n = \frac{(5n)((5n)+1)}{2} = \boxed{\frac{5n(5n+1)}{2}}.$$

2. **[2 points]** Suppose the numbers $1, 2, 3, \ldots, 3n$ are split into three sets of equal size with equal sum. Find a formula for the "magic sum" of each group. (For example, when $n = 3$, we can split $1, 2, 3 \ldots, 9$ into equal-sized sets A, B, C with $A = \{1, 6, 8\}$, $B = \{2, 4, 9\}$, and $C = \{3, 5, 7\}$. In this case, the "magic sum" is 15.)

The ~~All~~ numbers have total sum $1+2+\cdots+3n$, which equals

$$\frac{3n(3n+1)}{2}.$$

On the other hand, if s is the magic sum, then $3s$ is also the total sum of all numbers from 1 to $3n$.

Therefore $3s = \frac{3n(3n+1)}{2}$ and so $s = \boxed{\frac{n(3n+1)}{2}}$.

3. **[2 points]** Suppose that 20 lines are drawn in the plane. Assuming that no two lines are parallel and no three lines have a common intersection point, how many regions are there?

Starting from 1 region, we iteratively add lines.

Drawing the i^{th} line ~~one~~ increases the number of regions by i, as we have seen in class. So, ~~the total~~ if n lines are drawn the total number of regions is

$1 + \sum_{i=1}^{n} i$, which equals $1 + \frac{n(n+1)}{2}$.

With $n=20$, we have $1 + \frac{20(21)}{2}$ or $\boxed{211}$ regions.