Name: _____

Directions: Show all work unless directed otherwise. No credit for answers without work.

1. **[5 points]** Short Answer. Jesse and Marie use a simple Caeser cipher to exchange secret messages in social studies class. (They would never do such a thing in their mathematics classes.) You hear Jesse boast that their secret key k is the best possible key since it makes the processes of encoding and decoding exactly the same. Assuming that their encryption scheme is non-trivial (i.e. $k \neq 0$), what is k?

2. **[10 points]** Is the following statement true or false? Let a, b, and c be integers. If $a \mid b + c$, then $a \mid b$ and $a \mid c$. If the statement is true, give a proof. If the statement is false, give a counterexample (that is, a specific example of particular integers a, b, and c for which the statement fails).

3. **[5 points]** Find the quotient q and remainder r that results from dividing -308 by 86.

4. **[5 points]** Give 3 different examples of integers that are congruent to 79 modulo 145. If possible, one of your examples should be negative.

5. Proofs.

   (a) **[15 points]** Let a and b be integers. Prove that if there exist integers u and v such that $ua + vb = 1$, then $\gcd(a, b) = 1$.

   (b) **[15 points]** Let x and y be integers, not both zero, and let $d = \gcd(x, y)$. Prove that $\gcd(\frac{x}{d}, \frac{y}{d}) = 1$.

6. **[20 points]** Use the extended Euclidean algorithm to find $\gcd(28543, 32147)$ and express it as an integer combination of 28543 and 32147.

7. **[15 points]** Solve for x in $424x \equiv 19 \pmod{643}$.

8. The group of units.

   (a) **[5 points]** Give the multiplication table for $\mathbb{Z}_{10}^*$.

   (b) **[5 points]** What is $\phi(10)$?