Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. [**2 points**] Give a definition of $\text{ord}_p(n)$. (There are several equivalent ways to define this; give only one.)

2. [**2 points**] Find $\text{ord}_7(4900)$.

3. [**2 points**] Give 3 different examples of an integer n that satisfies $\text{ord}_2(n) = 3$.

4. [**2 points**] Let m be a positive integer and let p be a prime. For each algebraic structure below, circle the arithmetic operations that are well-behaved.

   (a) $\mathbb{Z}_m^*$ $\quad + \quad - \quad \times \quad \div$

   (b) $\mathbb{F}_p$ $\quad + \quad - \quad \times \quad \div$

   (c) $\mathbb{Z}_m$ $\quad + \quad - \quad \times \quad \div$

5. **[2 points]** Let a and b be positive integers such that $\mathrm{ord}_2(a) = \mathrm{ord}_2(b)$. Let k be the common order of 2 in both a and b; that is, $k = \mathrm{ord}_2(a)$ and $k = \mathrm{ord}_2(b)$. Prove that if $\mathrm{ord}_2(a+b) \geq k+1$.