Math373 Quiz 2 Fri. Jan 24, 2014

Name: Solutions

**Directions:** Show all work. No credit for answers without work.

1. **[1 point]** Determine whether the following statements are true or false. Write the entire word "true" or the entire word "false".

   (a) $15 \equiv 9 \pmod 6$ True

   (b) $6 \equiv -24 \pmod{10}$ True

   (c) $13 \equiv -6 \pmod 7$ FALSE

2. **[2 points]** Fill in the blanks: an integer $n$ is odd if and only if $n$ is congruent to 1 modulo 2.

3. **[3 points]** For which positive integers $m$ is it true that $17 \equiv 37 \pmod m$?

   $m \mid 17-37$ $m \mid -20$ $m \mid 20$

   Divisors of 20: $\boxed{1, 2, 4, 5, 10, 20}$

4. **[4 points]** Let $a$, $b$, and $c$ be integers, and let $m$ be a positive integer. Prove that if $a \equiv b \pmod m$ and $b \equiv c \pmod m$, then $a \equiv c \pmod m$.

Since $a \equiv b \bmod m$, $m \mid a-b$, so $a-b = um$ for some integer $u$.

Since $b \equiv c \bmod m$, $m \mid b-c$, so $b-c = vm$ for some integer $v$.

Adding these two equations gives

$$(a-b)+(b-c) = um+vm$$

or $a-c = (u+v)m$. Therefore $m \mid a-c$, and it follows that $a \equiv c \pmod m$.