Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 points]** In the divisibility lattice, which integer(s) are at the bottom? Which integer(s) are at the top?

bottom: ± 1

top: 0

2. **[4 points]** Let a, b, and m be integers, where $m \neq 0$. Prove that if the divisions $\frac{a}{m}$ and $\frac{b}{m}$ have the same remainder, then $m \mid a - b$.

Proof Since $\frac{a}{m}$ and $\frac{b}{m}$ have the same remainder r, it follows that

$$a = pm + r$$

$$b = qm + r$$

for some integers p and q. (These are the quotients of $\frac{a}{m}$ and $\frac{b}{m}$ respectively.) Subtracting these equations yields

$$a - b = (pm + r) - (qm + r)$$

or $a - b = (p - q)m$. Since $p - q$ is an integer, it follows that $m \mid a - b$. ☒

3. [4 points] Let $d = \gcd(56823, 2491)$. Use the extended Euclidean algorithm to find d and express d as an integer combination of 56823 and 2491.

$56823 = (22)\,2491 + 2021$

$2491 = (1)\,2021 + 470$

$2021 = (4)\,470 + 141$

$470 = (3)\,141 + 47$

$141 = (3)\,47 + 0$

So $\gcd(56,823, 2491) = 47$

$47 = 470 - (3)\cdot 141$

$= 470 - (3)(2021 - (4)470)$

$= (13)\,470 - (3)\,2021$

$= (13)(2491 - (1)2021) - (3)\,2021$

$= (13)(2491) - (16)(2021)$

$= (13)(2491) - (16)(56823 - (22)2491)$

$= \boxed{(365)(2491) - (16)(56823)}$.