Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. Modular Arithmetic Tables
   (a) Make addition and multiplication tables for $\mathbb{Z}_3$.
   (b) Make addition and multiplication tables for $\mathbb{Z}_6$.
2. Compute the following. Your answer should be an integer in the set $\{0, 1, \ldots, m-1\}$, where m is the modulus in the given problem.
   (a) $73 - 6173 \pmod{22}$
   (b) $342 \cdot 825 \pmod{17}$
3. Use the extended Euclidean algorithm to find a multiplicative inverse of 50 modulo 2891 in the range $\{0, 1, \ldots, 2890\}$.
4. Make a multiplication table for the unit group $\mathbb{Z}_9^*$.
5. Use the fast power algorithm to compute $2^{300} \pmod{1000}$. Show intermediate powers of 2.
6. Using a computer/calculator only for basic arithmetic operations (addition, subtraction, multiplication, and division), solve for x in $523x \equiv 211 \pmod{591}$. *Show your work.*
7. Let a, b, g, and m be integers such that $g^a \equiv 1 \pmod{m}$ and $g^b \equiv 1 \pmod{m}$. Prove that $g^{\gcd(a,b)} \equiv 1 \pmod{m}$.
8. **[Challenge]** Let a and b be integers, not both zero. The *least common multiple* of a and b, denoted $\text{lcm}(a, b)$, is the smallest positive integer that is a multiple of both a and b. For example, $\text{lcm}(10, 4) = 20$. Prove that $\text{lcm}(a, b) = (ab)/\gcd(a, b)$. *Hint:* find (and prove) a relationship between the positive common divisors of a and b and the positive common multiples of a and b.