Name: _____

Directions: This test has 6 pages, each worth 10 points. The test is scored out of 50 points. Your lowest scoring page is dropped. Unless explicitly stated, answers to counting problems do not need to be simplified.

1. [**3 points**] Find the numerical value of the coefficient of x^6y^9 in $(x+y)^{15}$.

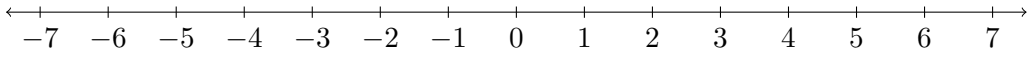
2. [**3 points**] Find the coefficient of $x^2y^6z^3$ in $(x+y+z)^{11}$.

3. [**4 points**] Find the coefficient of x^8 in $(5x-1)^{15}$. You do not need to simplify your answer.

4. [**5 points**] Write $\gcd(48995, 2855)$ as a linear combination of 48995 and 2855. Show your work.

5. [**2 parts, 1 point each**] Let S be the set of all linear combinations of 21 and 15.

   (a) On the number line below, circle the numbers that are in S.

   (b) Find a very simple description of the set S.

6. [**3 points**] The prime factorization of 14850 is given by $14850 = 2 \cdot 3^3 \cdot 5^2 \cdot 11$. Find $\varphi(14850)$.

7. [**2 parts, 5 points each**] In the RSA algorithm, let $p = 37$ and $q = 67$. Then $n = 2479$ and $\varphi(n) = 2376$. For the encryption key, pick $e = 17$.

   (a) Use the Euclidean algorithm to find the decryption key d.

   (b) Encode $T = 3$ using the public key (n, e).

8. [**2 parts, 2 points each**] Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Express the following permutations as the composition of zero or more disjoint cycles; each cycle should have at least 2 elements.

   (a) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 7 & 4 & 1 & 8 & 5 & 2 & 6 \end{pmatrix}$

   (b) $(6\ 8) \circ (2\ 7\ 6\ 4) \circ (4\ 1\ 2\ 8\ 7)$

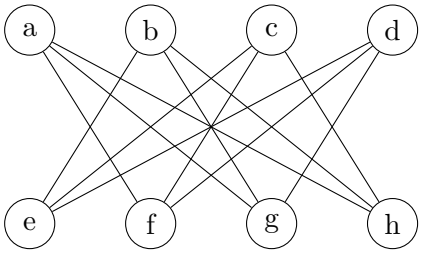
9. [**3 parts, 2 points each**] Let X be a set of size 10 and let $Y = \{1, 2, 3\}$.

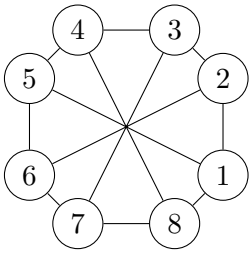
   (a) How many functions $f\colon X \to Y$ are there?

   (b) How many one-to-one/injective functions $f\colon X \to Y$ are there?

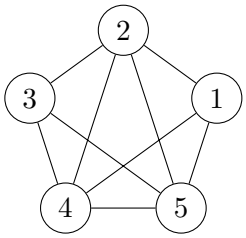
   (c) How many onto/surjective functions $f\colon X \to Y$ are there? *[Hint: For $j \in \{1, 2, 3\}$, let A_j be the set of all functions that map no elements in X to j. Count $|A_1 \cup A_2 \cup A_3|$ and use this to answer the question.]*

10. [**5 points**] Decide if the following graphs are isomorphic. If they are isomorphic, give the function that establishes the isomorphism. If not, explain why.





11. [**5 points**] Prove that the following graph is planar by finding a planar drawing.



12. (a) [**2 points**] What is the maximum number of edges possible in a simple planar graph with 100 vertices?

    (b) [**4 points**] Use part (a) to prove that if G is a simple planar graph on 100 vertices, then G has a vertex whose degree is at most 5.

13. [**2 points**] Draw the expression tree for $[(x \div 4) \cdot 3] + [(8 \cdot y) - (6 + x)]$.

14. [**2 points**] Draw the decision tree for sequential search on a list of three elements.