Name: _____

This test has 6 pages, each worth 10 points. The test is scored out of 50 points. Your lowest scoring page is dropped.

1. [**3 points**] Translate the following argument into a single wff. Hypotheses: If the refrigerator is cold, then it is plugged in. The refrigerator is unplugged if and only if it is quiet. The refrigerator is noisy. Conclusion: The refrigerator is cold. Use C for "the refrigerator is cold", P for "the refrigerator is plugged in", and Q for "the refrigerator is quiet".

2. Two parts.

   (a) [**6 points**] Write a truth table for the following wff:

   $$((A \wedge B) \leftrightarrow C) \vee \big(\big(C \rightarrow B'\big) \vee A\big)$$

   (b) [**1 point**] Is the wff a tautology?

Derivation Rule	Name/Abbreviation for Rule
$P \vee Q \iff Q \vee P$ $P \wedge Q \iff Q \wedge P$	Commutative—comm
$(P \vee Q) \vee R \iff P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \iff P \wedge (Q \wedge R)$	Associative—ass
$(P \vee Q)' \iff P' \wedge Q'$ $(P \wedge Q)' \iff P' \vee Q'$	De Morgan's laws—De Morgan
$P \to Q \iff P' \vee Q$	Implication—imp
$P \iff (P')'$	Double negation—dn
$P \leftrightarrow Q \iff (P \to Q) \wedge (Q \to P)$	Defn of Equivalence—equ
$\left.\begin{array}{l} P \\ P \to Q \end{array}\right\} \implies Q$	Modus ponens—mp
$\left.\begin{array}{l} P \to Q \\ Q' \end{array}\right\} \implies P'$	Modus tollens—mt
$\left.\begin{array}{l} P \\ Q \end{array}\right\} \implies P \wedge Q$	Conjunction—con
$P \wedge Q \implies \left\{\begin{array}{l} P \\ Q \end{array}\right.$	Simplification—sim
$P \implies P \vee Q$	Addition—add

3. [**10 points**] Using *only* the given derivation rules, give a proof sequence to show the following wff is a tautology.

$$(P \vee Q) \wedge P' \to Q$$

4. [**5 parts, 2 points each**] Determine whether the following sentences are valid. If the sentence is not valid, give an interpretation under which the sentence is false. Clearly indicate the domain of any interpretation you give.

   (a) $\exists x\,[Q(x)]$

   (b) $(\exists x\,[Q(x)] \to \forall x\,[Q(x)]) \to \exists x\,\left[(Q(x))'\right]$

   (c) $\forall x\,[\exists y\,[P(x,y)]] \to \exists x\,[\forall y\,[P(x,y)]]$

   (d) $\exists x\,[\forall y\,[P(x,y)]] \to \forall x\,[\exists y\,[P(x,y)]]$

   (e) $\exists x\,[\forall y\,[P(x,y)]] \to \forall y\,[\exists x\,[P(x,y)]]$

5. [**2 parts, 5 points each**] Prove the following statements.

   (a) The sum of three odd integers is odd.

   (b) If the product xy of two integers x and y is odd, then x is odd and y is odd.

6. [**10 points**] Prove by induction that $3^{3n} - 1$ is divisible by 13 for every positive integer n. *Hint*: do not multiply out large numbers. For example, do not replace $13 \cdot 15$ with 195.

7. [**10 points**] Prove by induction that any amount of postage greater than or equal to 12 cents can be made using only 3-cent and 5-cent stamps.