

Name: _____

1. [**2 parts, 1 point each**] A collection S of strings is defined recursively by

1. The empty string λ belongs to S .
 2. If X belongs to S , then bX and Xa belong to S .
- (a) Write down all the strings of length 4 that are in S .

(b) Give a simple, non-recursive definition of S that is equivalent to the given definition.

2. [**4 parts, 2 points each**] Solve the following recurrence relations.

(a)
$$T(n) = \begin{cases} 1 & n = 1 \\ T(n-1) + 2 & n \geq 2 \end{cases}$$

(b)
$$T(n) = \begin{cases} 1 & n = 1 \\ 3nT(n-1) & n \geq 2 \end{cases}$$

$$(c) \quad T(n) = \begin{cases} 1 & n = 1 \\ -3 & n = 2 \\ 5T(n-1) - 6T(n-2) & n \geq 3 \end{cases}$$

$$(d) \quad T(n) = \begin{cases} 2 & n = 1 \\ 1 & n = 2 \\ -2T(n-1) - T(n-2) & n \geq 3 \end{cases}$$