Math374 | Quiz #2 | 28 January 2011

Name: Key

1. [**4 parts, 1/2 point each**] Determine the truth value of each of the following wffs in the interpretation where the domain is $\{0, 1, 2, 3, \ldots\}$ and $P(x)$ is "x is even". (Remember that 0 is even.) Write the entire word "true" or the entire word "false".

   (a) $\forall x\,[\exists y\,[P(y) \wedge y > x]]$

   true

   (b) $\exists x\,[\forall y\,[P(y) \rightarrow y > x]]$

   false

   (Note: even if $x = 0$, it fails for $y = 0$)

   (c) $\exists x\,[\forall y\,[P(x) \rightarrow P(y)]]$

   true;

   (d) $\forall x\,[\exists y\,[P(x) \rightarrow P(y)]]$

   true

2. [**3 parts, 1 point each**] Using the predicates shown, translate the following into wffs.

$J(x)$: "x is a judge"	$C(x)$: "x is a chemist"	$L(x)$: "x is a lawyer"
$W(x)$: "x is a woman"	$A(x, y)$: "x admires y"	

   (a) No woman is both a lawyer and a chemist.

   $$\forall x\left[W(x) \rightarrow (L(x) \wedge C(x))'\right]$$

   Also OK: $\left(\exists x[W(x) \wedge L(x) \wedge C(x)]\right)'$

   (b) Every judge admires a female lawyer.

   $$\forall x\left[J(x) \rightarrow \exists y\left[W(y) \wedge L(y) \wedge A(x,y)\right]\right]$$

   (c) Some women admire only those who are lawyers or judges.

   $$\exists x\left[W(x) \wedge \forall y\left[A(x,y) \rightarrow L(y) \vee J(y)\right]\right]$$

3. [**2 points**] Write the negation of the sentence "Some lawyers admire only judges". Your sentence should be as simple as possible.

   Every lawyer admires someone who is not a judge.

Derivation Rule	Name/Abbreviation for Rule
$P \vee Q \iff Q \vee P$ $P \wedge Q \iff Q \wedge P$	Commutative—comm
$(P \vee Q) \vee R \iff P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \iff P \wedge (Q \wedge R)$	Associative—ass
$(P \vee Q)' \iff P' \wedge Q'$ $(P \wedge Q)' \iff P' \vee Q'$	De Morgan's laws—De Morgan
$P \to Q \iff P' \vee Q$	Implication—imp
$P \iff (P')'$	Double negation—dn
$P \leftrightarrow Q \iff (P \to Q) \wedge (Q \to P)$	Defn of Equivalence—equ
$\left.\begin{matrix} P \\ P \to Q \end{matrix}\right\} \implies Q$	Modus ponens—mp
$\left.\begin{matrix} P \to Q \\ Q' \end{matrix}\right\} \implies P'$	Modus tollens—mt
$\left.\begin{matrix} P \\ Q \end{matrix}\right\} \implies P \wedge Q$	Conjunction—con
$P \wedge Q \implies \left\{\begin{matrix} P \\ Q \end{matrix}\right.$	Simplification—sim
$P \implies P \vee Q$	Addition—add

4. [**3 points**] Using the given derivation rules and the 4 derivation rules involving quantifiers, give a proof sequence to show the following wff is valid.

$$\exists x\,[\forall y\,[Q(x,y)]] \to \forall y\,[\exists x\,[Q(x,y)]]$$

1.	$\exists x[\forall y[Q(x,y)]]$	hyp
2.	$\forall y[Q(a,y)]$	1, ei
3	$Q(a,y)$	2, ui
4.	$\exists x[Q(x,y)]$	3, eg
5.	$\forall y[\exists x[Q(x,y)]]$	4, ug