Name: Key

Show your work. Answers without work earn reduced credit.

1. [**7 parts, 1 point each**] Differentiate the following functions.

(a) $(5x^2+1)^6$

$$6(5x^2+1)^5\frac{d}{dx}[5x^2+1]$$

$$=6(5x^2+1)^5\cdot(10x+0)$$

$$=\boxed{60x(5x^2+1)^5}$$

(b) $y=\sqrt{s^3+1}$

$$\frac{d}{ds}\left[(s^3+1)^{1/2}\right]=\frac{1}{2}(s^3+1)^{-\frac{1}{2}}\cdot\frac{d}{ds}[s^3+1]$$

$$=\frac{1}{2\sqrt{s^3+1}}\cdot 3s^2=\boxed{\frac{3s^2}{2\sqrt{s^3+1}}}$$

(c) $f(x)=4x^2+\ln(x^2+1)$

$$f'(x)=8x+\frac{1}{x^2+1}\cdot\frac{d}{dx}[x^2+1]$$

$$=\boxed{8x+\frac{2x}{x^2+1}}$$

(d) $y=x\ln x$

$$\frac{d}{dx}[x\ln x]=\frac{d}{dx}[x]\ln x+x\frac{d}{dx}[\ln x]$$

$$=1\cdot\ln(x)+x\cdot\frac{1}{x}$$

$$=\boxed{\ln(x)+1}$$

(e) $y=(x+4)^6(x-1)^3$

$$\frac{dy}{dx}\left[(x+4)^6\right](x-1)^3+(x+4)^6\frac{d}{dx}\left[(x-1)^3\right]$$

$$=6(x+4)^5\frac{d}{dx}[x+4](x-1)^3+(x+4)^6\,3(x-1)^2\cdot\frac{d}{dx}[x-1]$$

$$=\boxed{6(x+4)^5(x-1)^3+(x+4)^6\,3(x-1)^2}$$

(f) $f(x)=(\ln(x)+e^{2x})^5$

$$f'(x)=5(\ln(x)+e^{2x})^4\cdot\frac{d}{dx}[\ln(x)+e^{2x}]$$

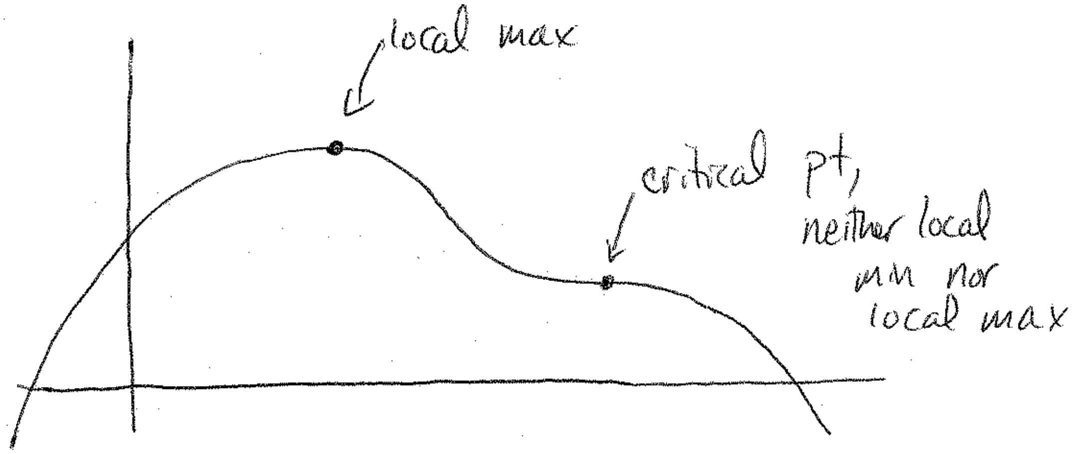
$$=\boxed{5(\ln(x)+e^{2x})^4\cdot\left(\frac{1}{x}+2e^{2x}\right)}$$

(g) $f(x)=\dfrac{x^3+2x}{x^2+1}$

$$f'(x)=\frac{(x^2+1)\frac{d}{dx}[x^3+2x]-(x^3+2x)\frac{d}{dx}[x^2+1]}{(x^2+1)^2}$$

$$=\boxed{\frac{(x^2+1)(3x^2+2)-(x^3+2x)\cdot 2x}{(x^2+1)^2}}$$

2. **[1 point]** Graph a function with two critical points. One of these critical points should be a local maximum, and the other should be neither a local maximum nor a local minimum.



3. **[2 parts, 1 point each]** Let $f(x) = x^2(x-5)^3$.

   (a) Find the critical points of $f(x)$.

$$f'(x) = \frac{d}{dx}[x^2](x-5)^3 + x^2\frac{d}{dx}[(x-5)^3]$$
$$= 2x(x-5)^3 + x^2 \cdot 3(x-5)^2 \cdot \cancel{\frac{d}{dx}[x-5]}\ 1$$
$$= 2x(x-5)^3 + 3x^2(x-5)^2$$
$$= x(x-5)^2[2(x-5)+3x] = x(x-5)^2(5x-10)$$

Critical pts:

$x=0$ or $(x-5)^2=0$ or $5x-10=0$

$x=5$ $\quad$ $x=2$

   (b) Use the First Derivative Test (i.e. sign chart) to classify the critical points. *Hint:* one of the critical points is neither a local minimum nor a local maximum.

f: ↗ increasing ↘ ↗ ↗

sign of f': + 0 − 0 + 0 +

0 $\quad$ 2 $\quad$ 5

$x(x-5)^2(5x-10)$: − + − $\quad$ + + − $\quad$ + + + $\quad$ + + +

local max $\quad$ local min $\quad$ neither