Name: Key

1. **[2 parts; 1.5 points each]** Differentiate the given function.

   (a) $f(x) = \ln(x^2+1)$

$$f'(x) = \frac{1}{x^2+1} \cdot \frac{d}{dx}\left[x^2+1\right]$$

$$= \frac{1}{x^2+1} \cdot 2x = \boxed{\frac{2x}{x^2+1}}$$

   (b) $f(x) = e^{\sqrt{x}}\ln(x)$

$$f'(x) = \frac{d}{dx}\left[e^{\sqrt{x}}\right]\ln(x) + e^{\sqrt{x}} \cdot \frac{d}{dx}\left[\ln(x)\right]$$

$$= e^{\sqrt{x}} \cdot \frac{d}{dx}\left[x^{1/2}\right] \cdot \ln(x) + e^{\sqrt{x}} \cdot \frac{1}{x}$$

$$= e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}\ln(x) + e^{\sqrt{x}} \cdot \frac{1}{x}$$

$$= \boxed{\frac{e^{\sqrt{x}}}{x}\left(\frac{1}{2}\sqrt{x}\ln(x) + 1\right)}$$

2. **[3 parts, 1 point each]** Find the indicated integral.

   (a) $\int 4x^5 - 2x^2\, dx$

$$= \int 4x^5\,dx - \int 2x^2\,dx$$

$$= 4\int x^5\,dx - 2\int x^2\,dx$$

$$= 4 \cdot \frac{1}{6}x^6 - 2 \cdot \frac{1}{3} \cdot x^3 + C$$

$$= \frac{2}{3}x^6 - \frac{2}{3}x^3 + C$$

$$= \boxed{\frac{2}{3}x^3(x^3 - 1) + C}$$

OVER →

(b) $\int \frac{1}{x^3}(x^2+1)^2\,dx$ $= \int \frac{1}{x^3}(x^4+2x^2+1)\,dx$

$= \int x + \frac{2}{x} + x^{-3}\,dx$

$= \int x\,dx + 2\int \frac{1}{x}dx + \int x^{-3}dx$

$= \boxed{\frac{x^2}{2} + 2\ln|x| - \frac{1}{2}x^{-2} + C}$

(c) $\int \frac{2t^2+3t+4}{t^{3/2}}\,dt$ $= \int 2t^{1/2} + 3t^{-1/2} + 4t^{-3/2}\,dt$

$= 2\int t^{1/2}dt + 3\int t^{-1/2}dt + 4\int t^{-3/2}dt$

$= 2\cdot\frac{2}{3}t^{3/2} + 3\cdot 2\cdot t^{1/2} + 4\cdot(-2)\cdot t^{-1/2} + C$

$= \boxed{\frac{4}{3}t^{3/2} + 6t^{1/2} - 8t^{-1/2} + C}$

3. **[2 points]** Solve the given initial value problem for $y = f(x)$.

$\frac{dy}{dx} = (e^x+1)^2$ where $y = \frac{1}{2}$ when $x = 0$.

$y = \int (e^x+1)^2\,dx = \int (e^x)^2 + 2e^x + 1\,dx$

$= \int e^{2x} + 2e^x + 1\,dx$

$= \int e^{2x}dx + 2\int e^x dx + \int 1\,dx$

$= \frac{1}{2}e^{2x} + 2e^x + x + C$

Solve for C: $\frac{1}{2} = \frac{1}{2}e^{2\cdot 0} + 2e^0 + 0 + C$

$\cancel{\frac{1}{2}} = \cancel{\frac{1}{2}\cdot 1} + 2\cdot 1 + C$

$C = -2$

OVER →

so $\boxed{y = \frac{1}{2}e^{2x} + 2e^x + x - 2}$

4. **[2 points]** Find the derivative $f'(x)$. (Hint: Use logarithmic differentiation.)

$$f(x) = \frac{(2x+3)^6(4x^2 - 3)^3}{\sqrt{x^5+2x}}$$

Note: I'm showing more steps than you need to show.

$$\ln(f(x)) = \ln\left(\frac{(2x+3)^6 \cdot (4x^2-3)^3}{(x^5+2x)^{1/2}}\right)$$

$$\ln(f(x)) = \ln\left((2x+3)^6\right) + \ln\left((4x^2-3)^3\right) - \ln\left((x^5+2x)^{\frac{1}{2}}\right)$$

$$\ln(f(x)) = 6\ln(2x+3) + 3\ln(4x^2-3) - \frac{1}{2}\ln(x^5+2x)$$

Diff both sides:

$$\frac{1}{f(x)} \cdot f'(x) = 6 \cdot \frac{1}{2x+3} \cdot 2 + 3 \cdot \frac{1}{4x^2-3} \cdot 8x - \frac{1}{2} \cdot \frac{1}{x^5+2x} \cdot (5x^4+2)$$

$$\frac{1}{f(x)} \cdot f'(x) = \frac{12}{2x+3} + \frac{24x}{4x^2-3} - \frac{5x^4+2}{2x^5+4x}$$

$$f'(x) = f(x) \cdot \left(\frac{12}{2x+3} + \frac{24x}{4x^2-3} - \frac{5x^4+2}{2x^5+4x}\right)$$

$$\boxed{f'(x) = \frac{(2x+3)^6(4x^2-3)^3}{\sqrt{x^5+2x}}\left(\frac{12}{2x+3} + \frac{24x}{4x^2-3} - \frac{5x^4+2}{2x^5+4x}\right)}$$