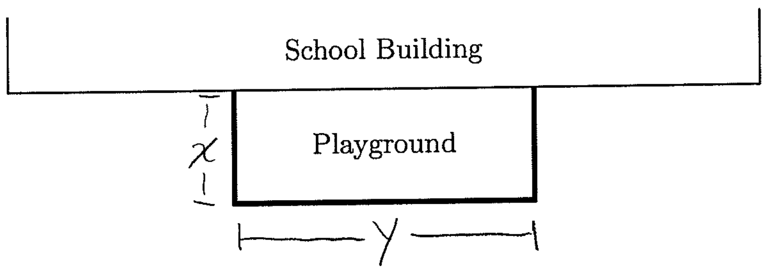
Name: Key

1. [~~2.5~~ 4 points] A school wishes to build a rectangular playground enclosed by a fence. The playground will be located along the side of the building, so that only 3 sides of the playground need fencing; these sides are shown in bold in the figure below. The school has 240 meters of fencing available. What is the largest possible area for the school's playground? 4



- $2x + y = 240$
- $y = 240 - 2x$
- Area $= x \cdot y$

$= x(240 - 2x)$

$= 240x - 2x^2$

- Maximize area for x in $[0, 120]$:

$A' = 240 - 4x$

$240 - 4x = 0$

$4x = 240$

$x = 60$

Check: $A(x)$ at $x = 0$, $x = 60$, $x = 120$:

$A(0) = 0 \cdot (240 - 0) = 0$

$A(60) = 60 \cdot (240 - 120)$

$= 6 \cdot 10 \cdot (120)$

$= 6 \cdot 12 \cdot 100 = 7200$

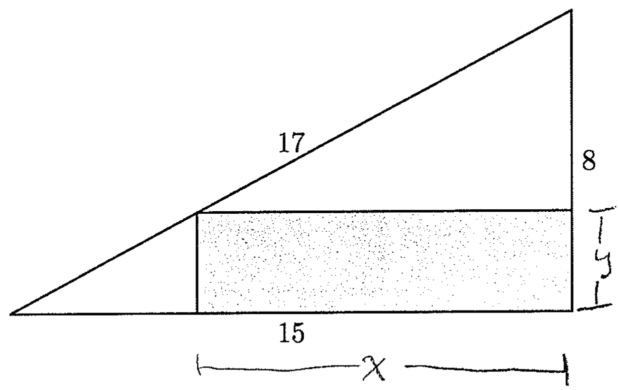
$A(120) = 0 \cdot (240 - 2 \cdot 120) = 0$

OVER →

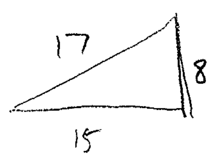
So the maximum area is $\boxed{7200 \text{ square meters}}$.

* Not on Actual Quiz, but good practice *

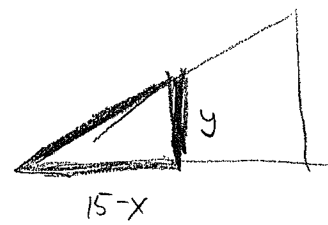
[~~2.5~~ 0 **points**] A rectangle is inscribed in a right triangle, as shown in the accompanying figure. If the triangle has sides of length 8, 15, and 17, what are the dimensions of the inscribed rectangle of greatest area?



- $A = xy$
- Similar triangles:



and



- $\dfrac{y}{8} = \dfrac{15-x}{15}$; $y = \frac{8}{15}(15-x) = 8 - \frac{8}{15}x$
- $A = xy = x\left(8 - \frac{8}{15}x\right) = 8x - \frac{8}{15}x^2$
- $A' = 8 - \frac{16}{15}x$
- Maximize A for x in $[0, 15]$:

$$0 = 8 - \frac{16}{15}x$$

$$\frac{16}{15}x = 8$$

$$x = 8 \cdot \frac{15}{16} = \frac{15}{2}$$

Check $A(x)$ at $x = 0, \frac{15}{2}, 15$:

- $A(0) = 0$, $A(15) = 0$
- $A\left(\frac{15}{2}\right) = \frac{15}{2} \cdot \left(8 - \frac{8}{15} \cdot \frac{15}{2}\right) = \frac{15}{2} \cdot (8-4) = 30$ ✓

OVER →

So dimensions of inscribed triangle of greatest area are $\boxed{\frac{15}{2} \text{ by } 4.}$

2/~~3~~. **[3 parts, 1 point each]** Use the exponentiation rules and logarithm rules to simplify the following.

(a) $(9^{3/2} - 16^{1/4})^{-1/2}$

$= ((9^{\frac{1}{2}})^3 - 2)^{-\frac{1}{2}}$

$= (3^3 - 2)^{-\frac{1}{2}}$

$= (27-2)^{-\frac{1}{2}}$

$= 25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \boxed{\frac{1}{5}}$

(b) $e^{3\ln 3 - 2\ln 5}$

$= e^{\ln 3^3 - \ln 5^2}$

$= e^{\ln 27 - \ln 25}$

$= e^{\ln\left(\frac{27}{25}\right)} = \boxed{\frac{27}{25}}$

(c) $\ln\left(\frac{x^2 y^{4/5}(z+2y)}{\sqrt{x^2+y^2+z^2}}\right)$

$= \ln(x^2 y^{4/5}(z+2y)) - \ln(\sqrt{x^2+y^2+z^2})$

$= \ln(x^2) + \ln(y^{4/5}) + \ln(z+2y) - \ln((x^2+y^2+z^2)^{1/2})$

$= \boxed{2\ln(x) + \frac{4}{5}\ln(y) + \ln(z+2y) - \frac{1}{2}\ln(x^2+y^2+z^2)}$

OVER →

3 ~~1~~. [1 point] Solve for x, simplifying as much as possible without using a calculator. Your final answer may involve e and/or logarithms.

$2 = e^{5x} + \cancel{1}\ 3/2$

$$e^{5x} = \frac{1}{2}$$

$$\ln(e^{5x}) = \ln\left(\frac{1}{2}\right)$$

$$5x = \ln(2^{-1})$$

$$5x = -\ln(2)$$

$$x = \boxed{-\frac{\ln(2)}{5}}$$

4 ~~5~~. [~~1~~ 2 point] How quickly will money double if it is invested at an annual interest rate of 5% compounded continuously? Simplify as much as possible without using a calculator; your final answer may involve e and/or logarithms.

$$B = Pe^{rt}$$

$$2P = Pe^{0.05t}$$

$$2 = e^{\frac{t}{20}}$$

$$\ln(2) = \ln\left(e^{\frac{t}{20}}\right)$$

$$\ln(2) = \frac{t}{20}$$

$$t = 20\ln(2)$$

So it will take $\boxed{20\ln(2)}$ years for the investment to double.