Math234 AD2/AD4 **Quiz #7** **5 March 2009**

Name: Key

1. **[2 points]** Find the horizontal and vertical asymptotes of the given function.

$$g\cancel{f}(x) = \frac{x+2}{x^2+x-2} = \frac{(x+2)}{(x+2)(x-1)}$$ Domain: all reals except $x=-2, x=1$.

Horiz: $$\lim_{x\to\infty} \frac{x+2}{x^2+x-2} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{2}{x^2}}{1+\frac{1}{x}-\frac{2}{x^2}} = \frac{0}{1} = 0$$

$$\lim_{x\to-\infty} \frac{x+2}{x^2+x-2} = 0$$

So g has a $\boxed{\text{horizontal asymptote } y=0}$

Vert: Candidates: $x=-2, x=1$.

$$\lim_{x\to-2} \frac{\cancel{x+2}}{(\cancel{x+2})(x-1)} = \lim_{x\to-2} \frac{1}{x-1} = -\frac{1}{3},$$ so $x=-2$ is not a vert. asymptote.

$$\left.\begin{aligned} \lim_{x\to1^-} \frac{1}{x-1} &= \frac{1}{\text{small neg}} = -\infty \\ \lim_{x\to1^+} \frac{1}{x-1} &= \frac{1}{\text{small pos}} = +\infty \end{aligned}\right\}$$ so g has a $\boxed{\text{vert. asymptote at } x=1}$.

2. **[4 parts, 2 points each]** Let $f(x) = (x^2-1)^2$. Note that because f is a polynomial, its domain is all real numbers and it does not have any asymptotes. You may find it useful to know that $\frac{1}{\sqrt{3}} \approx 0.577$.

(a) Find the y and x intercepts of f.

y-intercept:

$$y = (0^2-1)^2 = (-1)^2 = \boxed{1}$$

x-intercepts:

$$0 = (x^2-1)^2$$

$$\sqrt{0} = \sqrt{(x^2-1)^2}$$

$$0 = x^2-1$$

$$0 = (x+1)(x-1)$$

$$\boxed{x=-1} \text{ or } \boxed{x=1}$$

Recall $f(x) = (x^2-1)^2$.

(b) Compute the sign chart of f'. Use it to find the intervals of increase/decrease and relative extrema.

$f'(x) = 2(x^2-1)\cdot 2x = 4x(x^2-1)$. · $f'(x)$ is cont. everywhere

$4x(x^2-1) = 0$

$4x(x+1)(x-1) = 0$

$4x = 0$ or $x+1 = 0$ or $x-1 = 0$

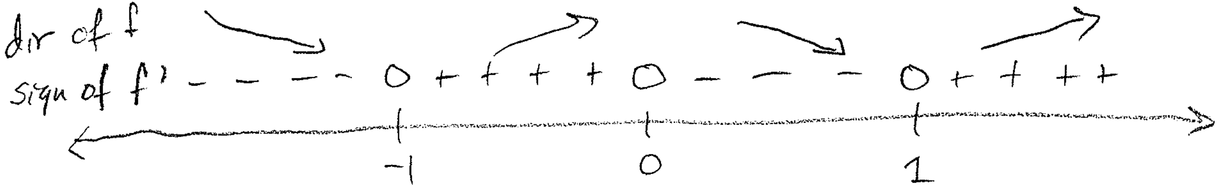
$x = 0$ or $x = -1$ or $x = 1$

$f'(-2)$ = neg · neg · neg = neg

$f'(-\frac{1}{2})$ = neg · pos · neg = pos

$f'(\frac{1}{2})$ = pos · pos · neg = neg

$f'(2)$ = pos · pos · pos = pos



Intervals of increase (if any):	$(-1, 0)$ and $(1, \infty)$
Intervals of decrease (if any):	$(-\infty, -1)$ and $(0, 1)$
Relative maxima (if any):	$(0, 1)$
Relative minima (if any):	$(-1, 0)$ and $(1, 0)$

(c) Compute the sign chart of f''. Use it to find the concavity of f and inflection points.

$$f''(x) = 4\left(1\cdot(x^2-1) + x\cdot 2x\right) = 4(3x^2-1)$$

‖ · $f''(x)$ is cont. everywhere

· $f''(-1) = 4(3-1) > 0$

· $f''(0) = -4 < 0$

· $f''(1) = 4(3-1) > 0$

$4(3x^2-1) = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

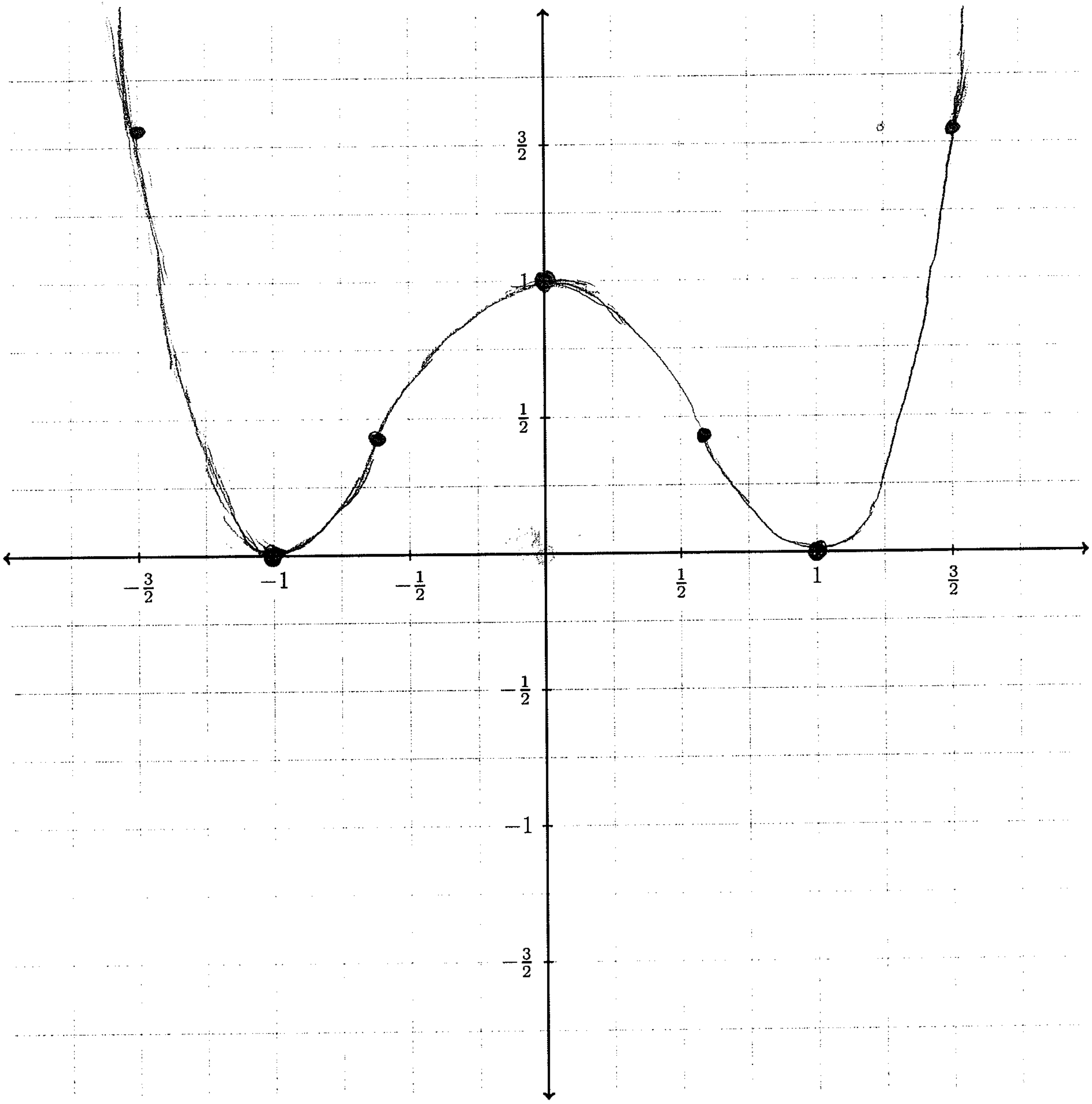
$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}} \approx \pm 0.577$

Concave up intervals (if any):	$(-\infty, -\frac{1}{\sqrt{3}})$ and $(\frac{1}{\sqrt{3}}, \infty)$
Concave down intervals (if any):	$(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
Inflection points (if any):	$(-\sqrt{\frac{1}{3}}, 4/9)$ and $(\sqrt{\frac{1}{3}}, \frac{4}{9})$

$f(-\frac{1}{\sqrt{3}}) = (\frac{1}{3}-1)^2 = (-\frac{2}{3})^2 = \frac{4}{9}$

$f(\frac{1}{\sqrt{3}}) =$ " " $= \frac{4}{9}$

(d) Using parts (a)-(c), sketch the graph of f below.



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