Math234 AD2/AD4 **Quiz #6** **26 February 2009**

Name: Key

1. [~~4~~ 5 **points**] Find the intervals of increase/decrease, and the x-coordinates of relative extrema for the following function.

$$f(x) = \frac{2x^3}{x-4}$$

$$\cdot\ f'(x) = \frac{(x-4)\cdot 6x^2 - 2x^3\cdot(1)}{(x-4)^2}$$

$$= \frac{6x^3 - 24x^2 - 2x^3}{(x-4)^2} = \frac{4x^3-24x^2}{(x-4)^2} = \frac{4x^2(x-6)}{(x-4)^2}$$

∘ Note: $f'(x)$ is continuous everywhere except $x=4$.

$\cdot\ f'(x)=0:$

$$\frac{4x^2(x-6)}{(x-4)^2}=0$$

$$4x^2(x-6)=0$$

$$x^2=0 \quad \text{or} \quad x-6=0$$

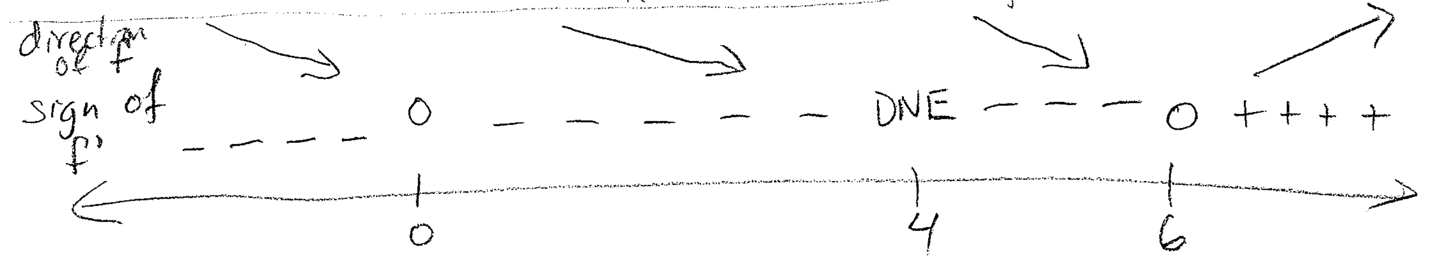
$$x=0 \quad \text{or} \quad x=6$$

$\cdot\ f'(-1) = \frac{4\cdot 1\cdot -7}{\text{pos}} < 0$

$\cdot\ f'(1) = \frac{4\cdot 1\cdot -5}{\text{pos}} < 0$

$\cdot\ f'(5) = \frac{4\cdot\text{pos}\cdot\text{neg}}{\text{pos}} < 0$

$\cdot\ f'(7) = \frac{4\cdot\text{pos}\cdot\text{pos}}{\text{pos}} > 0$



Intervals of increase (if any):	$(6, \infty)$
Intervals of decrease (if any):	$(-\infty, 0)$, $(0, 4)$, and $(4, 6)$
Relative maxima (if any):	none
Relative minima (if any):	$x=6$

OVER →

2. [~~4~~ 5 **points**] Find the equation of the tangent line to the given curve at the specified point.
$y^2x + \frac{2x}{y} = 5; \qquad (1,2)$

Tangent line:

$$\boxed{y - 2 = -\frac{10}{7}(x-1)}$$

$$\frac{d}{dx}\left[y^2x + \frac{2x}{y}\right] = \frac{d}{dx}[5]$$

$$\frac{d}{dx}[y^2x] + \frac{d}{dx}[2xy^{-1}] = 0$$

$$2y\cdot\frac{dy}{dx}\cdot x + y^2\cdot 1 + 2\left[y^{-1} + x(-1)y^{-2}\cdot\frac{dy}{dx}\right] = 0$$

$$2yx\frac{dy}{dx} + y^2 + \frac{2}{y} - \frac{2x}{y^2}\cdot\frac{dy}{dx} = 0$$

$$2yx\frac{dy}{dx} - \frac{2x}{y^2}\cdot\frac{dy}{dx} = -y^2 - \frac{2}{y}$$

$$\left(2yx - \frac{2x}{y^2}\right)\frac{dy}{dx} = -y^2 - \frac{2}{y}$$

$$\frac{dy}{dx} = \frac{-y^2 - \frac{2}{y}}{2yx - \frac{2x}{y^2}} \cdot \frac{-y^2}{-y^2} = \frac{y^4 + 2y}{2x - 2y^3x}$$

$$m_{tan} = \left(\frac{dy}{dx}\right)\bigg|_{(1,2)} = \frac{2^4 + 4}{2\cdot1 - 2\cdot2^3\cdot1} = \frac{20}{2-16} = -\frac{20}{14} = -\frac{10}{7}$$

3. [~~2~~ **points**] Use the derivative to estimate how much the function $f(x) = x^2 + 3x + 5$ changes from $x = 2$ to $x = 2.1$.

Not on actual quiz but
Still good practice:

$\cdot\, f'(x) = 2x + 3$

$$\cdot\, f(2.1) - f(2) = \frac{f(2+0.1) - f(2)}{0.1}\cdot 0.1$$

$$\approx f'(2)\cdot 0.1$$

$$= 7\cdot 0.1 = \boxed{0.7}$$