Name: Key

1. Differentiate the given function.

   (a) **[2 points]** $f(x) = (3x^8 - 10x^2 + 4)^5$

$$f'(x) = 5(3x^8 - 10x^2 + 4)^4 \cdot \frac{d}{dx}\left[3x^8 - 10x^2 + 4\right]$$

$$= 5(3x^8 - 10x^2 + 4)^4 \cdot (24x^7 - 20x)$$

$$= \boxed{20x(3x^8 - 10x^2 + 4)^4(6x^6 - 5)}$$

   (b) **[3 points]** $f(x) = \sqrt{\dfrac{x^2+1}{2x+5}}$

$$f(x) = \sqrt{(x^2+1)(2x+5)^{-1}}$$

$$= \left((x^2+1)(2x+5)^{-1}\right)^{1/2}$$

$$= (x^2+1)^{1/2}(2x+5)^{-1/2}$$

$$f'(x) = \frac{d}{dx}\left[(x^2+1)^{1/2}\right]\cdot(2x+5)^{-1/2} + (x^2+1)^{1/2}\frac{d}{dx}\left[(2x+5)^{-1/2}\right]$$

$$= \left(\tfrac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x\right)(2x+5)^{-\frac{1}{2}} + (x^2+1)^{1/2}\left(-\tfrac{1}{2}(2x+5)^{-3/2}\cdot 2\right)$$

$$= \boxed{x(x^2+1)^{-\frac{1}{2}}(2x+5)^{-\frac{1}{2}} - (x^2+1)^{1/2}(2x+5)^{-3/2}}$$

OVER →

$$\cancel{= (x^2+1)^{1/2}\left[x(2x+5)^{-\frac{1}{2}} - (2x+5)^{-3/2}\right]}$$

$$\cancel{= (x^2+1)^{1/2}(2x+5)^{-3/2}\left[x(2x+5) - 1\right]}$$

$$\cancel{= (x^2+1)^{1/2}(2x+5)^{-3/2}\left[2x^2+5x-1\right]}$$

2. (a) **[3 points]** Differentiate the given function. Simplify your answer.
$f(x) = 5(6-x)^4(2x-1)^3$

$$f'(x) = 5\left(\frac{d}{dx}\left[(6-x)^4\right](2x-1)^3 + (6-x)^4\frac{d}{dx}\left[(2x-1)^3\right]\right)$$

$$= 5\left(4(6-x)^3\cdot(-1)(2x-1)^3 + (6-x)^4\,3(2x-1)^2(2)\right)$$

$$= 5(6-x)^3\left(-4(2x-1)^3 + 6(6-x)(2x-1)^2\right)$$

$$= 5(6-x)^3(2x-1)^2\left(-4(2x-1) + 6(6-x)\right)$$

$$= 10(6-x)^3(2x-1)^2\left(-2(2x-1) + 3(6-x)\right)$$

$$= \boxed{10(6-x)^3(2x-1)^2(-7x+20)}$$

(b) **[2 points]** Find all values of x at which the tangent line is horizontal.

$$f'(x) = 0$$

$$10(6-x)^3(2x-1)^2(-7x+20) = 0$$

$(6-x)^3 = 0$ or $(2x-1)^2 = 0$ or $-7x+20 = 0$

$((6-x)^3)^{1/3} = 0^{1/3}$ | $2x-1 = 0$ | $x = \frac{20}{7}$

$6-x = 0$ | $x = \frac{1}{2}$

$x = 6$

So $f(x)$ has a horizontal tangent line at $\boxed{x=\frac{1}{2},\ x=\frac{20}{7},\ \text{and } x=6}$