Name: Key

1. **[3 parts, 2 points each]** Differentiate the given function. ~~Simplify your answers.~~

(a) $f(x) = 2x^4 - 3x^2 + x - 8$

$$f'(x) = 2\cdot 4x^3 - 3\cdot 2x + 1\cdot x^0$$
$$= \boxed{8x^3 - 6x + 1}$$

(b) $f(x) = \dfrac{3}{x^2} - 2\sqrt{x} + \dfrac{1}{\sqrt{4x}} + x^{-4.1} + \sqrt{5}$

$$= 3x^{-2} - 2x^{\frac{1}{2}} + \frac{1}{2\sqrt{x}} + x^{-4.1} + \sqrt{5}$$
$$= 3x^{-2} - 2x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + x^{-4.1} + \sqrt{5}$$
$$\boxed{f'(x) = -6x^{-3} - x^{-\frac{1}{2}} - \frac{1}{4}x^{-\frac{3}{2}} - 4.1x^{-5.1}}$$

(c) $f(x) = \dfrac{x^3 + 6}{\sqrt{x} - 1}$

$$f'(x) = \frac{\text{low}\cdot d(\text{hi}) - \text{hi}\cdot d(\text{low})}{(\text{low})^2}$$

$$= \frac{(\sqrt{x} - 1)\left(\frac{d}{dx}(x^3+6)\right) - (x^3+6)\cdot\left(\frac{d}{dx}(\sqrt{x} - 1)\right)}{(\sqrt{x} - 1)^2}$$

$$= \boxed{\frac{(\sqrt{x} - 1)(3x^2) - (x^3+6)\cdot\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{x - 2\sqrt{x} + 1}}$$

OVER →

2. **[2 points]** Decide if the following function is continuous at the specified value of x, and explain why.

$$f(x) = \begin{cases} x^2+1 & \text{if } x<3 \\ 8 & \text{if } x=3 \\ 2x+4 & \text{if } x>3 \end{cases} \quad \text{at } x=3$$

$\boxed{\text{No}}$, $f(x)$ is not continuous at $x=3$, because condition (3) fails:

(1) $f(c)$ is defined $(f(3)=8)$. ✓

(2) $\lim_{x\to3} f(x)$ exists. $\left(\lim_{x\to3} f(x) = 10\right)$ ✓

(3) $\lim_{x\to3} f(x) = f(c)$ (Fails because $8 \neq 10$). X

3. **[2 points]** Find an equation for the tangent line to the given curve at the point where $x = x_0$.
$y = (2\sqrt{x}+5x)(x^2-1)$; $x_0 = 1$

$$\frac{dy}{dx} = (2\sqrt{x}+5x)\cdot\frac{d}{dx}(x^2-1) + \left[\frac{d}{dx}(2\sqrt{x}+5x)\right]\cdot(x^2-1)$$

$$= (2\sqrt{x}+5x)\cdot(2x) + (x^{-\frac{1}{2}}+5)(x^2-1)$$

$$m_{tan} = \left(\frac{dy}{dx}\right)\Big|_{x=1} = (2\sqrt{1}+5(1))\cdot(2(1)) + (1^{-\frac{1}{2}}+5)\underbrace{\cancel{(1^2-1)}}_{\to 0}$$

$$= 7 \cdot 2$$

$$= 14.$$

- Tangent line contains the point $(1, y(1)) = (1, 0)$.
- Tangent line: $y - y_1 = m_{tan}(x - x_1)$

$$\boxed{\begin{aligned} y - 0 &= 14(x-1) \\ y &= 14x - 14 \end{aligned}}$$