Math234 AD2/AD4 Quiz #12 23 April 2009

Name: Key

1. **[1 point]** Compute the indicated function value.
$f(x,y) = (x - y^2)^3 - \frac{xy}{x+y} \quad f(6,3)$

$$f(6,3) = (6-3^2)^3 - \frac{6\cdot 3}{6+3}$$

$$= (6-9)^3 - \frac{18}{9}$$

$$= (-3)^3 - 2 = -27 - 2 = \boxed{-29}$$

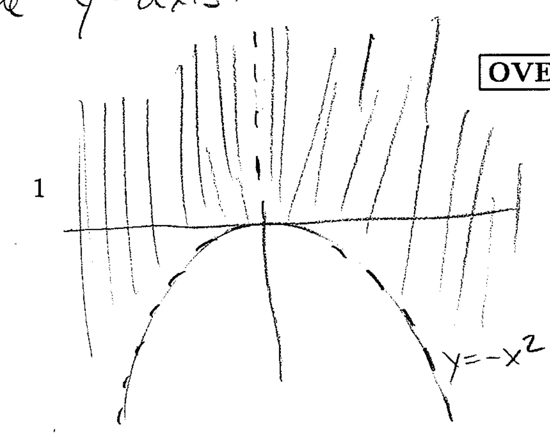
2. **[1 point]** Describe the domain of the function $g(x,y) = \frac{\ln(x^2+y)}{x}$.

- All pts $(x,y)$ where $x \neq 0$, and $x^2+y>0$
$y > -x^2$.

Algebraic: $\boxed{\text{Domain is all pts } (x,y) \text{ where } x \neq 0 \text{ and } y > -x^2}$

Geometric: Domain is all pts $(x,y)$ strictly above the parabola $y=-x^2$ and not on the $y$-axis:

**OVER →**



3. **[4 parts, 1 point each]** Let $f(x, y) = x^2 - 2xy^2 + y^3$.

(a) Find the first partials $f_x(x, y)$ and $f_y(x, y)$.

$$f_x(x,y) = \frac{\partial}{\partial x}\left[x^2 - 2xy^2 + y^3\right] = \boxed{2x - 2y^2} = \boxed{2(x-y^2)}$$

$$f_y(x,y) = \frac{\partial}{\partial y}\left[\underset{\text{const}}{x^2} - 2xy^2 + y^3\right] = -2x\frac{\partial}{\partial y}\left[y^2\right] + \frac{\partial}{\partial y}\left[y^3\right]$$

$$= -2x \cdot 2y + 3y^2$$

$$= -4xy + 3y^2$$

$$= \boxed{y(3y - 4x)}$$

(b) Find the critical point(s) of $f(x, y)$.

$f_x = 0$:

$$2(x-y^2) = 0$$
$$x - y^2 = 0$$
$$x = y^2.$$

$f_y = 0$, using $x = y^2$:

$$y(3y-4x) = 0$$
$$y(3y-4y^2) = 0$$
$$y^2(3-4y) = 0$$
$$y^2 = 0 \quad \text{or} \quad 3 - 4y = 0$$
$$y = 0 \quad \text{or} \quad {}^{2}\ 4y = 3$$
$$y = 3/4.$$

$y=0$: $x = y^2 = 0^2 = 0$, so $(0,0)$ is a critical pt.

$y = 3/4$: $x = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$, so $\left(\frac{9}{16}, \frac{3}{4}\right)$ is a critical pt.

Crit pts: $\boxed{(0,0) \text{ and } \left(\frac{9}{16}, \frac{3}{4}\right)}$

OVER →

(c) Find the second partials $f_{xx}(x,y)$, $f_{xy}(x,y)$, $f_{yx}(x,y)$, and $f_{yy}(x,y)$.

- $f_{xx}(x,y) = \frac{\partial}{\partial x}\left[2(x-y^2)\right] = 2\frac{\partial}{\partial x}\left[x-y^2\right] = 2\cdot 1 = \boxed{2}$
- $f_{xy}(x,y) = \frac{\partial}{\partial y}\left[2(x-y^2)\right] = 2\frac{\partial}{\partial y}\left[x-y^2\right] = 2\cdot(-2y) = \boxed{-4y}$
- $f_{yx}(x,y) = \frac{\partial}{\partial x}\left[y(3y-4x)\right] = y\frac{\partial}{\partial x}\left[3y-4x\right] = y(-4) = \boxed{-4y}$

same ✓

- $f_{yy}(x,y) = \frac{\partial}{\partial y}\left[y(3y-4x)\right] = \frac{\partial}{\partial y}\left[3y^2-4xy\right] = \boxed{6y-4x}$

(d) Where possible, use the second partials test to classify the critical point(s) of $f(x,y)$.

- $D(x,y) = f_{xx}(x,y)\,f_{yy}(x,y) - \left(f_{xy}(x,y)\right)^2.$

In fact, this point is a saddle point.

- $D(0,0) = 2\cdot(6\cdot 0 - 4\cdot 0) - (-4\cdot 0)^2$

$= 0 - 0 = 0,$ $\boxed{\text{so test is inconclusive at } (0,0).}$

- $D\left(\frac{9}{16},\frac{3}{4}\right) = 2\cdot\left(6\cdot\frac{3}{4} - 4\cdot\frac{9}{16}\right) - \left(-4\cdot\frac{3}{4}\right)^2$

$= 2\cdot\left(\frac{9}{2}-\frac{9}{4}\right) - (-3)^2$

$= 9 - \frac{9}{2} - 9 = -\frac{9}{2} < 0,$

$\boxed{\text{So } \left(\frac{9}{16},\frac{3}{4}\right) \text{ is a saddle point.}}$

**OVER →**

4. **[2 points]** Find the first-order partial derivatives of $g(x,y) = \sqrt{x}e^{xy^2}$.

$$\cdot\, g_x(x,y) = \frac{\partial}{\partial x}\left[x^{\frac{1}{2}}e^{xy^2}\right] = \frac{\partial}{\partial x}\left[x^{\frac{1}{2}}\right]e^{xy^2} + x^{\frac{1}{2}}\frac{\partial}{\partial x}\left[e^{xy^2}\right]$$

$$= \frac{1}{2}x^{-\frac{1}{2}}e^{xy^2} + x^{\frac{1}{2}}e^{xy^2}\cdot\frac{\partial}{\partial x}\left[xy^2\right]$$

$$= \frac{e^{xy^2}}{2\sqrt{x}} + \sqrt{x}\,e^{xy^2}\cdot y^2$$

$$= \boxed{e^{xy^2}\left(\frac{1}{2\sqrt{x}} + \sqrt{x}\,y^2\right)}$$

$$\cdot\, g_y(x,y) = \frac{\partial}{\partial y}\left[x^{\frac{1}{2}}e^{xy^2}\right] = x^{\frac{1}{2}}\frac{\partial}{\partial y}\left[e^{xy^2}\right] = \sqrt{x}\,e^{xy^2}\cdot\frac{\partial}{\partial y}\left[xy^2\right]$$

$$= \sqrt{x}e^{xy^2}\cdot 2xy = \boxed{2x^{3/2}ye^{xy^2}}$$

5. **[2 points]** Use the linear approximation of the given function at $(2,4)$ to approximate $f(1.8, 4.1)$.

$f(x,y) = \dfrac{xy}{y+1}$

$$\cdot\, f_x(x,y) = \frac{1}{y+1}\frac{\partial}{\partial x}\left[xy\right] = \frac{y}{y+1}$$

$$\cdot\, f_y(x,y) = x\frac{\partial}{\partial y}\left[\frac{y}{y+1}\right] = x\cdot\frac{(y+1)\cdot 1 - y}{(y+1)^2} = \frac{x}{(y+1)^2}$$

$$\cdot\, \Delta x = 1.8 - 2 = -0.2 = -\frac{1}{5}$$

$$\cdot\, \Delta y = 4.1 - 4 = 0.1 = \frac{1}{10}.$$

$$\cdot\, \Delta z = f_x(2,4)\Delta x + f_y(2,4)\Delta y$$

$$= \frac{4}{4+1}\left(-\frac{1}{5}\right) + \frac{2}{(4+1)^2}\cdot\frac{1}{10}$$

$$= -\frac{4}{25} + \frac{2}{250}$$

$$= -\frac{40}{250} + \frac{2}{250} = -\frac{38}{250} = \frac{-19}{125}$$

$$\cdot\, f(1.8, 4.1) \approx f(2,4) + \Delta z$$

$$= \frac{2\cdot 4}{5} + \left(-\frac{19}{125}\right)$$

$$= \frac{8\cdot 25}{5\cdot 25} - \frac{19}{125}$$

$$= \frac{200-19}{125}$$

$$= \boxed{\frac{181}{125}}$$