Name: Key

1. **[2 parts; 2.5 points each]** Find the indicated integral.

(a) $\int \sqrt{3x+5}\,dx$

$u = 3x+5$

$du = 3dx$

$dx = \frac{1}{3}du$

$$\int \sqrt{3x+5}\,dx = \int \sqrt{u}\cdot\frac{1}{3}du$$
$$= \frac{1}{3}\int u^{1/2}\,du$$
$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{9}(3x+5)^{3/2} + C}$$

(b) $\int \frac{8x^3+2}{(x^4+x)^3}\,dx$

$u = x^4+x$

$du = (4x^3+1)dx$

$dx = \frac{1}{4x^3+1}du$

$$\int \frac{8x^3+2}{(x^4+x)^3}dx = \int \frac{8x^3+2}{u^3}\cdot\frac{1}{4x^3+1}du$$
$$= \int \frac{2\cancel{(4x^3+1)}}{u^3}\cdot\frac{1}{\cancel{4x^3+1}}du$$
$$= 2\int \frac{1}{u^3}du$$
$$= 2\int u^{-3}du$$
$$= 2\cdot\frac{1}{-2}u^{-2} + C$$
$$= -\frac{1}{u^2} + C$$
$$= \boxed{-\frac{1}{(x^4+x)^2} + C}$$

OVER →

2. [**2 parts; 2.5 points each**] Evaluate the definite integral.

(a) $\int_0^{1/2} \frac{t}{2t+1}\, dt$

$\int \frac{t}{2t+1}\, dt = \int \frac{t}{u} \cdot \frac{1}{2}\, du$

$u = 2t+1$ | $u-1 = 2t$

$du = 2\,dt$ | $\frac{u-1}{2} = t$

$dt = \frac{1}{2}\, du$

$= \frac{1}{2}\int \frac{1}{u} \cdot \frac{u-1}{2}\, du$

$= \frac{1}{4}\int \frac{u-1}{u}\, du$

$= \frac{1}{4}\int \frac{u}{u} - \frac{1}{u}\, du$

$= \frac{1}{4}\int 1 - \frac{1}{u}\, du$

$= \frac{1}{4}\left(u - \ln|u|\right) + C$

$= \frac{1}{4}\left(2t+1 - \ln|2t+1|\right) + C$

$\int_0^{1/2} \frac{t}{2t+1}\, dt = \frac{1}{4}\left(2t+1 - \ln|2t+1|\right)\Big|_0^{1/2}$

$= \frac{1}{4}\left(2 - \ln|2|\right) - \frac{1}{4}\left(1 - \ln|1|\right)$

$= \frac{2}{4} - \frac{\ln(2)}{4} - \frac{1}{4}(1-0)$

$= \frac{1}{4} - \frac{\ln(2)}{4}$

$= \boxed{\frac{1-\ln(2)}{4}}$

(b) $\int_{\cancel{0}\, e}^{e^6} \frac{\sqrt{\ln(x)+3}}{x}\, dx$

$u = \ln(x) + 3$

$du = \frac{1}{x}\, dx$

$\int \frac{\sqrt{\ln(x)+3}}{x}\, dx$

$= \int \sqrt{\ln(x)+3} \cdot \frac{1}{x}\, dx$

$= \int \sqrt{u}\, du$

$= \frac{2}{3} u^{3/2} + C$

$= \frac{2}{3}\left(\ln(x)+3\right)^{3/2} + C$

$\int_0^{e^6} \frac{\sqrt{\ln(x)+3}}{x}\, dx = \left(\frac{2}{3}\left(\ln(x)+3\right)^{3/2}\right)\Big|_e^{e^6}$

$= \frac{2}{3}\left(\ln(e^6)+3\right)^{3/2} - \frac{2}{3}\left(\ln(e)+3\right)^{3/2}$

$= \frac{2}{3}(6+3)^{3/2} - \frac{2}{3}(1+3)^{3/2}$

$= \frac{2}{3}\left(9^{\frac{1}{2}}\right)^3 - \frac{2}{3}\left(4^{\frac{1}{2}}\right)^3$

$= \frac{2}{3}(3^3) - \frac{2}{3}(2^3)$

$= 2 \cdot 3^2 - \frac{2}{3} \cdot 8$

$= 2\left(9 - \frac{8}{3}\right)$

$= 2\left(\frac{27}{3} - \frac{8}{3}\right) = 2\left(\frac{19}{3}\right) = \boxed{\frac{38}{3}} = \boxed{12 + \frac{2}{3}}$