Key

1. **[4 parts, 1 point each]** Compute the indicated values of the given function; simplify as much as possible.

$$f(t) = \begin{cases} 1 & \text{if } t < -12 \\ 2t - 3 & \text{if } -12 \le t < 1 \\ t^{3/2} & \text{if } t \ge 1 \end{cases}$$

(a) $f(-20) = \boxed{1}$

(b) $f(-12) = 2(-12) - 3$
$= -24 - 3$
$= \boxed{-27}$

(c) $f(1/2) = 2(\frac{1}{2}) - 3$
$= 1 - 3 = \boxed{-2}$

(d) $f(4) = (4)^{3/2} = (4^{1/2})^3$
$= (\sqrt{4})^3$
$= 2^3 = \boxed{8}$

2. **[2 points]** Determine the domain of the given function.

$f(z) = \dfrac{z-2}{\sqrt{25 - z^2}}$

square root: $25 - z^2 \geq 0$ required, so $z^2 \leq 25$ and $-5 \leq z \leq 5$ is required.

can't divide by 0: $\sqrt{25 - z^2} \neq 0$ required, so $z \neq \pm 5$ required.

Therefore, the domain is $\boxed{(-5, 5)}$.

3. **[2 points]** Find the composite function $f(g(x))$. Simplify as much as possible.

$f(u) = (u-1)^3$, $g(x) = x^4 + 1$

$f(g(x)) = f(x^4+1) = ((x^4+1) - 1)^3$
$= (x^4)^3$
$= \boxed{x^{12}}$

4. **[2 points]** Find the difference quotient of f; namely, $\dfrac{f(x+h) - f(x)}{h}$. Simplify as much as possible.

$f(x) = x^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$$= \boxed{2x + h}$$