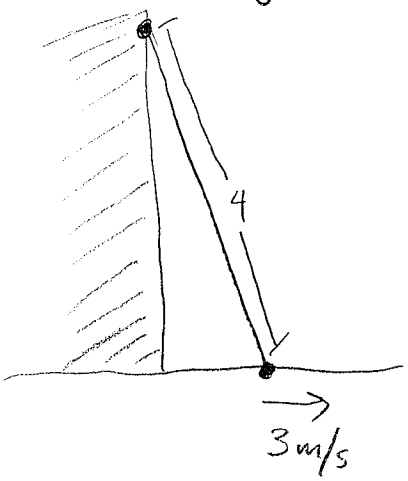
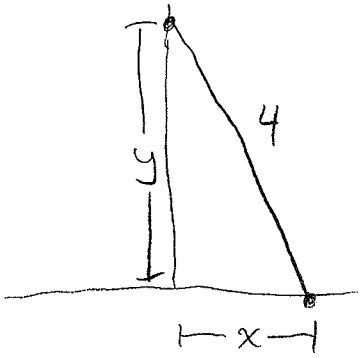
Related Rates

A 4 meter ladder is propped against a building and begins to slide. The point in contact with the ground slides at 3 meters per second. How fast is the point of contact with the building moving down when the bottom of the ladder is 2m from the building?



Soln:



- Pythagorean Thm: $x^2 + y^2 = 4^2$

  $x^2 + y^2 = 16$

- View x and y as functions of time: $x(t)$ and $y(t)$.
- Know: $x'(t) = 3$
- Want to know: $y'(t)$ when $x = 2$.
- Differentiate both sides of $x^2 + y^2 = 16$ with respect to t:

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}\left[16\right]$$

$$2xx' + 2yy' = 0$$

$$2yy' = -2xx'$$

$$y' = \frac{-2xx'}{2y}$$

$$y' = \frac{-x}{y}x'$$

$$y' = -\frac{2}{y}\cdot 3$$

- Still need to know y when $x = 2$. Use

$$x^2 + y^2 = 16$$

$$2^2 + y^2 = 16$$

$$y^2 = 12$$

$$y = \pm\sqrt{12}$$, throw away negative soln.

$$y = \sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}.$$

- So when $x = 2$:

$$y' = -\frac{2}{y}\cdot 3$$

$$= -\frac{2}{2\sqrt{3}}\cdot 3$$

$$= -\frac{3}{\sqrt{3}}$$

$$= -\sqrt{3}$$

Therefore the top of the ladder moves down at $\boxed{\sqrt{3}\text{ m/s}}$.