Linear Approximations of Functions in two variables

- Use the linear approx of $f(x,y) = x^2y + \frac{1}{xy}$ at $(2,3)$ to approximate $f(2.2, 2.9)$.

Soln:

- $f_x(x,y) = \frac{\partial}{\partial x}[x^2y] + \frac{\partial}{\partial x}\left[\frac{1}{xy}\right]$

$= y\frac{\partial}{\partial x}[x^2] + \frac{1}{y}\frac{\partial}{\partial x}[x^{-1}]$

$= y \cdot 2x + \frac{1}{y}\cdot(-1)x^{-2}$

$= 2xy - \frac{1}{x^2y}$

- $f_y(x,y) = \frac{\partial}{\partial y}[x^2y] + \frac{\partial}{\partial y}\left[\frac{1}{xy}\right]$

$= x^2 + \frac{1}{x}\frac{\partial}{\partial y}[y^{-1}]$

$= x^2 + \frac{1}{x}(-1)y^{-2}$

$= x^2 - \frac{1}{xy^2}$

- $\Delta x = 2.2 - 2 = 0.2 = \frac{1}{5}$
- $\Delta y = 2.9 - 3 = -0.1 = -\frac{1}{10}$

<u>So:</u>

$$\Delta z = f_x(x,y)\,\Delta x + f_y(x,y)\,\Delta y$$

$$= f_x(2,3)\left(\tfrac{1}{5}\right) + f_y(2,3)\left(-\tfrac{1}{10}\right)$$

$$= \left(2\cdot 2\cdot 3 - \frac{1}{2^2\cdot 3}\right)\left(\frac{1}{5}\right) + \left(2^2 - \frac{1}{2\cdot 3^2}\right)\left(-\frac{1}{10}\right)$$

$$= \left(12 - \frac{1}{12}\right)\left(\frac{1}{5}\right) + \left(4 - \frac{1}{18}\right)\left(-\frac{1}{10}\right)$$

$$= \frac{143}{12}\cdot\frac{1}{5} + \frac{71}{18}\left(-\frac{1}{10}\right)$$

$$= \frac{143}{60} - \frac{71}{180}$$

$$= \frac{143\cdot 3}{180} - \frac{71}{180} = \frac{429-71}{180} = \frac{358}{180} = \frac{179}{90}.$$

<u>And</u>: $f(2.2, 2.9) \approx f(2,3) + \Delta z$

$$= 2^2\cdot 3 + \frac{1}{2\cdot 3} + \frac{179}{90}$$

$$= 12 + \frac{1}{6} + \frac{179}{90} = \boxed{\frac{637}{45}}$$

- Numbers on a quiz or exam would probably be a bit friendlier.