

Name: Solutions**Directions:** Solve the following problems. Give supporting work/justification where appropriate.

1. [3 parts, 1 point each] Translate the following sentences to symbolic logic as directly and simply as possible. Is the statement true or false? Write the entire word.

(a) The product of two real numbers is zero if and only if one of the factors is zero.

$$\boxed{\forall x, y \in \mathbb{R}, xy = 0 \iff x = 0 \vee y = 0}$$

True, this is a basic fact about multiplication.

(b) Every integer divided by a non-zero integer is an integer.

$$\boxed{\forall x, y \in \mathbb{Z}, y \neq 0 \Rightarrow \frac{x}{y} \in \mathbb{Z}} \quad \text{or} \quad \boxed{\forall x, y \in \mathbb{Z}, y \neq 0 \Rightarrow \exists z \in \mathbb{Z}, z = \frac{x}{y}}$$

False, for example if $x=3$ and $y=2$, then $\frac{3}{2}$ is not an integer.

(c) There is no maximum integer.

$$\boxed{\neg(\exists m \in \mathbb{Z}, \forall x \in \mathbb{Z}, x \leq m)} \quad \text{or} \quad \boxed{\forall m \in \mathbb{Z}, \exists x \in \mathbb{Z}, x > m}$$

This is true. If m is an integer, then $m+1$ is a larger integer.

2. [3 parts, 1 point each] Translate the following statements in symbolic logic to English sentences as simply as possible. Is the statement true or false? Write the entire word.

(a) $\forall x, y, z \in \mathbb{R}, (x < y) \wedge (y < z) \implies x < z$

less than is a transitive relation on the real numbers or True

for all real numbers x, y , and z , if $x < y$ and $y < z$ then $x < z$.

(b) $\forall x, y \in \mathbb{Z}, (\exists k \in \mathbb{Z}, x = 2k \vee y = 2k) \implies (\exists l \in \mathbb{Z}, x + y = 2l)$

The sum of two integers, at least one of which is even, is even.

This is false; for example, if $x=2$ and $y=3$, then $x+y$ is odd.

(c) $\forall A \subseteq \mathbb{Q}, (\exists s, t \in \mathbb{Q}, \forall a \in A, s \leq a \leq t) \implies (\exists n \in \mathbb{N}, |A| \leq n)$

Every bounded set of rational numbers is finite

This is false; for example, $\{a \in \mathbb{Q} : 0 \leq a \leq 1\}$ is an infinite set but all its members are between 0 and 1 (and hence it is a bounded set).

3. [2 points] Give an English sentence which asserts the negation of the sentence P below as naturally as possible. P : Either there are finitely many primes or every infinite set of positive integers contains at least one prime.

~P: There are infinitely many primes and there is an infinite set of positive integers that contains no primes.

4. [2 points] Prove that the sum of two even integers is even.

Pf Let x and y be even integers. Since x and y are even, we have that $x = 2k_1$ and $y = 2k_2$ for some $k_1, k_2 \in \mathbb{Z}$. We compute

$$x + y = 2k_1 + 2k_2 = 2(k_1 + k_2).$$

Since $k_1 + k_2$ is an integer, it follows that $x + y$ is also even. $\square$