Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[4 parts, 1 point each]** Let $A = \{1, 2\}$, $B = \varnothing$, and $C = \{3\}$. Find the following sets.

   (a) $\mathcal{P}(A)$

   $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

   (b) A^2

   $\{(1,1), (1,2), (2,1), (2,2)\}$

   (c) $\mathcal{P}(B^0)$

   $B^0 = \{()\} = \{a\}$, where $a = ()$.

   $\mathcal{P}(B^0) = \mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$

   $= \boxed{\{\emptyset, \{()\}\}}$

   (d) $A \times C \times A$

   $\{(1,3,1), (1,3,2), (2,3,1), (2,3,2)\}$

2. **[1 point]** Is it true that $\mathbb{R}^3 = \mathbb{R}^2 \times \mathbb{R}$? Explain why or why not.

   No, it is not true. A typical element in $\mathbb{R}^3$ has the form (x, y, z), but a typical element in $\mathbb{R}^2 \times \mathbb{R}$ has the form $((x, y), z)$.

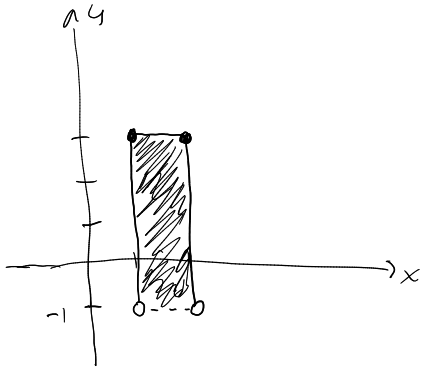
3. **[1 point]** Let $D = \{1, 2, 3, 4\}$. Express $\{X \subseteq D : |D| = 2\}$ by listing the elements between braces.

   With the correction $\{X \subseteq D : |X| = 2\}$

   $\{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}\}$

   As originally written:
   → Since $|D| = 4$ and $4 = 2$ is false no matter what X is, no set X satisfies the membership condition, so this set equals $\boxed{\{\}}$.

4. **[1 point]** Draw a picture of $[1,2] \times (-1,3]$ in the plane. Use solid lines to indicate boundaries in the set and dashed lines to indicate boundaries outside the set.



$[1,2] \times (-1,3] = \{(x,y) : 1 \leq x \leq 2 \text{ and } -1 < y \leq 3\}$

5. **[6 parts, 0.5 points each]** True/False. Write the entire word true or false. No justification necessary. We define:

$A = \{1, 2, \{1,2\}, \{\varnothing\}\}$ $B = \{1, \{1,1\}, \varnothing\}$ $C = \{\{1\}\}$ $D = \{\varnothing\}$

(a) $1 \in A$

TRUE

$A = \{1, 2, \{1,2\}, \varnothing\}$, $1 \in A$

(c) $1 \subseteq B$

FALSE 1 is not a set

(e) $D \in B$

FALSE. Although $\varnothing \in B$, $\{\varnothing\}$ is not an element in B.

(b) $C \in A$

FALSE:

$\{\{1\}\}$ is not one of the elts in A

(d) $C \subseteq B$

TRUE C has one element, namely $\{1\}$, and it is a member of B since $\{1,1\} = \{1\}$.

(f) $D \subseteq C$

FALSE. D has an element, namely $\varnothing$, which is not an element of C.