

Name: Solutions**Directions:** Solve the following problems. Give supporting work/justification where appropriate.

1. [4 parts, 1 point each] Decide whether the following statements are true or false, writing the entire word "true" or "false" to indicate your answer. Give brief explanations.

(a) $\{1, 2, 3\} = \{1, 1, 2, 3, 3\}$

True; both sides are the set whose elements are 1, 2, and 3.

(b) $\{1, 2, 3\} = \{3, 2, 1\}$

True; sets do not care about order.

(c) $1 \in \{0, 0.9, 0.99, 0.999, 0.9999, \dots\}$

False. Every element in the set is less than 1.

(d) $\{\{1\}, \{1, 1\}\} = \{\{1\}\}$

True. Both sides are the singleton set whose member is $\{1\}$.

2. [4 parts, 1 point each] Determine whether the following sets are infinite or finite. If the set is finite, then determine its cardinality.

(a) $\{1, \{1\}, \{\{1\}\}, \{\{\{1\}\}\}, \dots\}$

Infinite. The elements $1, \{1\}, \{\{1\}\}, \{\{\{1\}\}\}, \dots$ are all distinct.

(b) $\{\mathbb{R}\}$

Finite. This is a set with 1 element, namely the set of real numbers.

(c) $\{x \in \mathbb{R} : x^2 = 1\}$

Finite. This set is $\{-1, 1\}$, so it has cardinality 2. Indeed, $x^2 = 1$ is equivalent to $x^2 - 1 = 0$, or $(x+1)(x-1) = 0$.

(d) $\{1, 2, 3, \{1, 2\}, 1, \{2, 1, 2\}\}$

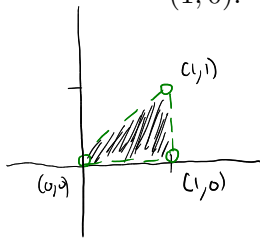
Finite. This set has size 4; it contains the integers 1, 2, and 3, and the set $\{1, 2\}$. Note that $\{1, 2\} = \{2, 1, 2\}$.

3. [2 parts, 1 point each] Use set-builder notation to express the following sets compactly.

(a) $\{0, 0.9, 0.99, 0.999, 0.9999, \dots\}$

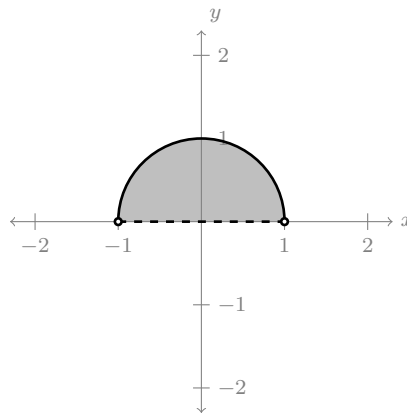
$$\left\{ 1 - \frac{1}{10^n} : n \in \mathbb{Z} \text{ and } n \geq 0 \right\}$$

(b) The set of all points (x, y) in the interior of the triangle with vertices $(0, 0)$, $(1, 1)$, and $(1, 0)$.



$$\{(x, y) \in \mathbb{R}^2 : 0 < x < 1 \text{ and } 0 < y < x\}$$

4. [2 points] Use set-builder notation to express the subset of $\mathbb{R}^2$ displayed below.



$$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ and } y > 0\}$$