

Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. See “Guidelines and advice” on the course webpage for more information.

1. [BP 1.8.{6,8,10}] Recall that for real numbers a and b , we have $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$. Find the following sets.

(a) $\bigcup_{i \in \mathbb{N}} [0, i + 1]$	(d) $\bigcap_{\alpha \in \mathbb{R}} (\{\alpha\} \times [0, 1])$
(b) $\bigcap_{i \in \mathbb{N}} [0, i + 1]$	(e) $\bigcup_{x \in [0, 1]} ([x, 1] \times [0, x^2])$
(c) $\bigcup_{\alpha \in \mathbb{R}} (\{\alpha\} \times [0, 1])$	(f) $\bigcap_{x \in [0, 1]} ([x, 1] \times [0, x^2])$

2. For each $k \in \mathbb{N}$, let $A_k = \{kn : n \in \mathbb{Z}\}$. Find the following sets.

(a) The examples A_1 , A_2 , and A_3 .	(d) $\bigcap_{k=1}^3 A_k$
(b) $\bigcup_{k=1}^3 A_k$	(e) $\bigcap_{k=1}^{\infty} A_k$
(c) $\bigcup_{k=2}^4 A_k$	(f) $\bigcup_{k \in I} A_k$ where $I = \{3, 5, 7, 9, 11, \dots\}$.

3. [BP 2.1, evens] Decide whether or not the following are statements. In the case of a statement, say if it is true or false, if possible.

- (a) Every even integer is a real number.
- (b) Sets $\mathbb{Z}$ and $\mathbb{N}$.
- (c) Some sets are finite.
- (d) $\mathbb{N} \notin \mathcal{P}(\mathbb{N})$.
- (e) $(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R}) = \mathbb{N} \times \mathbb{N}$.
- (f) If the integer x is a multiple of 7, then it is divisible by 7.
- (g) Call me Ishmael.
- (h) If x is an integer, then $x + y$ is also an integer.

4. [BP 2.2] Express each statement or open sentence in one of the forms $P \wedge Q$, $P \vee Q$, or $\sim P$. Be sure to state explicitly what P and Q mean in each part.

- (a) The matrix A is not invertible.
- (b) $x < y$
- (c) The quiz is scheduled for Wednesday or Friday.
- (d) $x \in A \cap B$

5. An infinite series of nested circles and squares are drawn, all sharing a common center point. The outermost circle has radius 1. The space between each circle and the square it circumscribes is shaded. What is the total area of the shaded regions?

