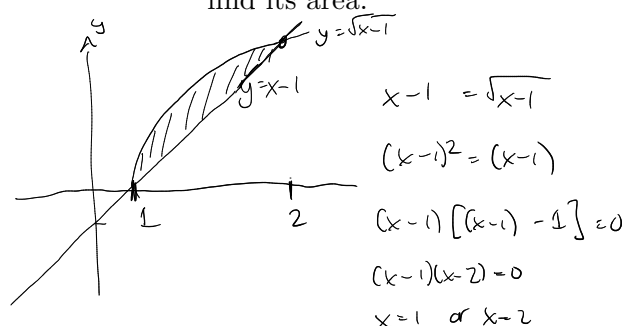


Directions:

1. Write your name with one character in each box below.
2. Show all work. No credit for answers without work.
3. All answers should be simplified.

1. [4 points] Sketch the region enclosed by the curves $y = \sqrt{x-1}$ and $x - y = 1$ and find its area.

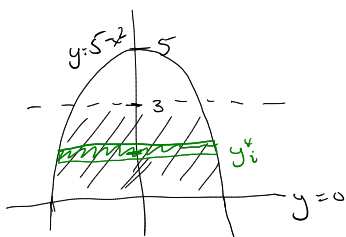


$$\begin{aligned}
 \text{Area} &= \int_1^2 \sqrt{x-1} - (x-1) dx & u &= x-1 \\
 & & du &= dx \\
 &= \int_0^1 \sqrt{u} - u du \\
 &= \left(\frac{2}{3} u^{3/2} - \frac{1}{2} u^2 \right)_{u=0}^{u=1} \\
 &= \left(\frac{2}{3} (1)^{3/2} - \frac{1}{2} (1) \right) - (0 - 0) = \frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \boxed{\frac{1}{6}}
 \end{aligned}$$

2. [3 points] Find the average value of the function $f(x) = xe^{x^2}$ in the interval $[1, 4]$.

$$\begin{aligned}
 f_{\text{ave}} &= \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{4-1} \int_1^4 x e^{x^2} dx & u &= x^2 \\
 & & du &= 2x dx \\
 &= \frac{1}{3} \int_1^4 e^{x^2} \cdot \frac{1}{2} \cdot 2x dx = \frac{1}{3} \int_1^{16} e^u \cdot \frac{1}{2} du = \frac{1}{6} \int_1^{16} e^u du = \frac{1}{6} (e^u)_{u=1}^{u=16} \\
 &= \frac{1}{6} (e^{16} - e^1) = \boxed{\frac{1}{6} (e^{16} - e)}
 \end{aligned}$$

3. [3 points] A parabolic dam is bounded by the equations $y = 5 - x^2$ and $y = 0$, where x and y are in meters. The water level is at $y = 3$. Given that Force = (Pressure)(Area), Pressure = $\delta \cdot \text{Depth}$ where $\delta = 9800 \text{ N/m}^3$, give a definite integral for the force due to hydrostatic pressure on the dam. **Do not solve the integral.**



$$\begin{aligned}
 y &= 5 - x^2 \\
 x^2 &= 5 - y \\
 x &= \pm \sqrt{5 - y}
 \end{aligned}$$

Force of water in strip = Pressure $\cdot$ Area

$$= (\delta \cdot (3 - y_i^*)) (2\sqrt{5 - y_i^*}) dy$$

$$\text{Total Force} = \int_0^3 \delta(3-y) 2\sqrt{5-y} dy = \boxed{2\delta \int_0^3 (3-y)\sqrt{5-y} dy}$$

$$= 19,600 \int_0^3 (3-y)\sqrt{5-y} dy$$

Not Required: To solve: $u = 5 - y$, $y = 5 - u$
 $du = -dy$

$$= -2\delta \int_0^3 (3-y)\sqrt{5-y} (-1) dy = -2\delta \int_5^2 (u-2) u^{1/2} du$$

$$= 2\delta \int_2^5 u^{3/2} - 2u^{1/2} du = 2\delta \left(\frac{2}{5} u^{5/2} - 2 \cdot \frac{2}{3} u^{3/2} \right) \Big|_{u=2}^{u=5} \approx 2\delta(8.962) \approx 175,656 \text{ N}$$