Directions:

1. Write your name with one character in each box below.
2. Show all work. No credit for answers without work.
3. All answers should be simplified.

1. **[6 parts, 2 points each]** Use the u-substitution to compute the following.

(a) $\displaystyle\int_1^3 \frac{\ln x}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$= \int_1^3 \ln x \cdot \frac{1}{x}\,dx$

$= \int_0^{\ln 3} u\,du$

$= \left(\frac{1}{2}u^2\right)\Big|_{u=0}^{u=\ln 3} = \frac{1}{2}\left((\ln 3)^2 - 0^2\right)$

$= \boxed{\frac{1}{2}(\ln 3)^2}$

(b) $\displaystyle\int x(x^2+1)^4\,dx$

$u = x^2+1$
$du = 2x\,dx$

$= \int (x^2+1)^4 \cdot \frac{1}{2} \cdot 2x\,dx$

$= \int u^4 \cdot \frac{1}{2} \cdot du = \frac{1}{2}\int u^4\,du$

$= \frac{1}{2}\left(\frac{1}{5}u^5\right) + C = \frac{1}{10}u^5 + C$

$= \boxed{\frac{1}{10}(x^2+1)^5 + C}$

(c) $\displaystyle\int \sec^2 x \tan^2 x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$= \int \tan^2 x \cdot \sec^2 x\,dx$

$= \int u^2\,du = \frac{1}{3}u^3 + C$

$= \frac{1}{3}(\tan x)^3 + C$

$= \boxed{\frac{1}{3}\tan^3 x + C}$

(d) $\displaystyle\int \frac{x^2}{1+x^6}\,dx$

$u = x^3$
$du = 3x^2\,dx$

$= \int \frac{1}{1+x^6} \cdot \frac{1}{3} \cdot 3x^2\,dx$

$= \int \frac{1}{1+(x^3)^2} \cdot \frac{1}{3} \cdot 3x^2\,dx$

$= \int \frac{1}{1+u^2} \cdot \frac{1}{3} \cdot du$

$= \frac{1}{3}\int \frac{1}{1+u^2}\,du$

$= \frac{1}{3}\arctan(u) + C$

$= \boxed{\frac{1}{3}\arctan(x^3) + C}$

(e) $\displaystyle\int x^3(x^2+1)^4\,dx$

$u = x^2+1, \quad x^2 = u-1$
$du = 2x\,dx$

$= \int x^2(x^2+1)^4 \cdot \frac{1}{2} \cdot 2x\,dx$

$= \int (u-1)\,u^4 \cdot \frac{1}{2} \cdot du$

$= \frac{1}{2}\int (u-1)u^4\,du$

$= \frac{1}{2}\int u^5 - u^4\,du$

$= \frac{1}{2}\left(\frac{u^6}{6} - \frac{u^5}{5}\right) + C$

$= \boxed{\frac{1}{12}(x^2+1)^6 - \frac{1}{10}(x^2+1)^5 + C}$

(f) $\displaystyle\int \frac{1}{x^2-2x+5}\,dx$

$= \int \frac{1}{(x-1)^2+4}\,dx$

$= \int \frac{\frac{1}{4}}{\frac{1}{4}\left[(x-1)^2+4\right]}\,dx$

$= \frac{1}{4}\int \frac{1}{\left(\frac{x-1}{2}\right)^2+1}\,dx$

$u = \frac{x-1}{2}$
$du = \frac{1}{2}\,dx$

$= \frac{1}{4}\int \frac{1}{\left(\frac{x-1}{2}\right)^2+1} \cdot 2 \cdot \frac{1}{2}\,dx$

$= \frac{1}{4}\int \frac{1}{u^2+1} \cdot 2 \cdot du$

$= \frac{1}{2}\int \frac{1}{1+u^2}\,du$

$= \frac{1}{2}\arctan(u) + C$

$= \boxed{\frac{1}{2}\arctan\left(\frac{x-1}{2}\right) + C}$