

Directions:

1. Write your name with one character in each box below.
2. Show all work. No credit for answers without work.

1. **[15 points]** Solve the following system of equations.

$$\begin{array}{rrrrrrcl} x_1 & - & x_2 & - & x_3 & = & 0 \\ -30x_1 & + & 36x_2 & - & 7x_3 & = & 7 \\ -5x_1 & + & 6x_2 & - & x_3 & = & 1 \end{array}$$

2. **[15 points]** Give an equation for the components of $\mathbf{b}$ that determines when the system $A\mathbf{x} = \mathbf{b}$ is consistent.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 7 \\ -2 & -2 & -14 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

3. **[15 points]** List the elementary row operations and give a brief description of each.

4. **[15 points]** Determine which values of h , if any, make the linear system represented by the following augmented matrix have infinitely many solutions.

$$\left[\begin{array}{cccc} 3 & -2 & 5 & 1 \\ 2 & h & 2 & -1 \\ 1 & -1 & 3 & 2 \end{array} \right]$$

5. [15 points] Given the augmented matrix, express the solution set in parametric form.

$$\left[\begin{array}{ccccccccc} 1 & 5 & 0 & 0 & -7 & 4 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 6 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{array} \right]$$

6. [5 parts, 2 points each] True/False. Justify your answers.
- (a) Every matrix is row-equivalent to infinitely many matrices.
 - (b) Every inconsistent linear system can be made consistent by deleting a carefully chosen equation.
 - (c) If $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{c}$ are consistent systems, then the solution sets are translations of one another.
 - (d) If $\mathbf{a}_1, \dots, \mathbf{a}_n \in \mathbb{R}^m$ and $n \geq m$, then $\text{Span}(\{\mathbf{a}_1, \dots, \mathbf{a}_n\}) = \mathbb{R}^m$.
 - (e) If $\mathbf{u}$ is a scalar multiple of $\mathbf{a}$ and $\mathbf{v}$ is a scalar multiple of $\mathbf{b}$, then $\mathbf{u} + \mathbf{v}$ is a scalar multiple of $\mathbf{a} + \mathbf{b}$.

7. An economy has 3 sectors: tech, food, and energy. The output of the tech sector is consumed as follows: $4/5$ to tech, $1/10$ to food, and $1/10$ to energy. The output of the food sector is consumed as follows: $1/3$ to tech, $1/6$ to food, and $1/2$ to energy. The output of the energy sector is consumed as follows: $1/4$ to tech, $1/4$ to food, and $1/2$ to energy.

(a) **[10 points]** Let p_t , p_f , and p_e , be the total cost (equivalently, the total value) in billions of the food, energy, and tech sectors, respectively. Assuming that the total cost (or total value) of each sector equals the that sector's total expenses, obtain a linear system in variables p_t , p_f , and p_e .

(b) **[5 points]** Given that the total value of the economy is \$120 billion (i.e. $p_t + p_f + p_e = 120$), find the value of each sector. (Hint: first scale each row to eliminate fractions and then try to avoid reintroducing them.)

