Name: ______________________________

Directions: Show all work. No credit for answers without work.

1. [**4 parts, 1 point each**] Determine whether the following vectors are linearly independent or linearly dependent. Justify your answer.

(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ -6 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -7 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 0 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}, \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$

2. [**3 points**] Determine the values of h that make the following vectors linearly independent.

$$\begin{bmatrix} 2 \\ 14 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ h \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 35 \\ h+1 \end{bmatrix}$$

3. **[2 parts, 1 point each]** True/False. Justify your answers.

   (a) If the columns of A are linearly independent, then $A\mathbf{x} = \mathbf{0}$ has a unique solution.

   (b) If $A\mathbf{x} = \mathbf{b}$ has a unique solution for at least one vector $\mathbf{b}$, then the columns of A are linearly independent.

4. **[1 point]** Prove that if $\mathbf{x}$ and $\mathbf{y}$ are linearly independent but $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z} \in \text{Span}\{\mathbf{x}, \mathbf{y}\}$.