Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [**4 points**] Solve the following linear system.

$$\begin{array}{rcrcrcl} & & 2x_2 & + & x_3 & = & 1 \\ -2x_1 & + & 17x_2 & + & 16x_3 & = & 1 \\ x_1 & - & 8x_2 & - & 8x_3 & = & 1 \end{array}$$

$$\begin{bmatrix} 0 & 2 & 1 & 1 \\ -2 & 17 & 16 & 1 \\ 1 & -8 & -8 & 1 \end{bmatrix} \xrightarrow{R1 \leftrightarrow R3} \begin{bmatrix} 1 & -8 & -8 & 1 \\ -2 & 17 & 16 & 1 \\ 0 & 2 & 1 & 1 \end{bmatrix} \xrightarrow{R2 \pm 2(R1)} \begin{bmatrix} 1 & -8 & -8 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 1 & 1 \end{bmatrix} \xrightarrow{R3 \pm (-2)R2} \begin{bmatrix} 1 & -8 & -8 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

$$\xrightarrow{R1 \pm 8R3} \begin{bmatrix} 1 & -8 & 0 & -39 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{bmatrix} \xrightarrow{R1 \pm 8R2} \begin{bmatrix} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

So the unique soln is given by

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -15 \\ 3 \\ -5 \end{bmatrix}.$$

2. [**3 points**] Give an example of an **inconsistent** linear system with two equations Eq1 and Eq2 such that each equation individually has infinitely many solutions.

Many solutions are possible, such as

$$\boxed{\begin{array}{l} x_1 + x_2 = 0 \\ x_1 + x_2 = 1 \end{array}}$$

3. **[3 points]** The following augmented matrix represents a linear system. Find all values for h that make the system consistent. (Hint: simplify the second and third rows as much as possible before involving h in your computation. Avoid fractions if possible.)

$$\begin{bmatrix} -1 & h & 30 \\ 5 & -2 & -24 \\ 3 & 2 & 8 \end{bmatrix}$$

$= 10 - \frac{96}{32}$

$= 7$

$\frac{224}{32} = \frac{320 - 100 + 4}{32}$

$$\begin{array}{l} R1 \cdot (-1) \\ R3 \cdot 2 \end{array} \begin{bmatrix} 1 & -h & -30 \\ 5 & -2 & -24 \\ 6 & 4 & 16 \end{bmatrix} \xrightarrow{R3 \pm (-1)R2} \begin{bmatrix} 1 & -h & -30 \\ 5 & -2 & -24 \\ 1 & 6 & 40 \end{bmatrix} \xrightarrow{R1 \leftrightarrow R3} \begin{bmatrix} 1 & 6 & 40 \\ 5 & -2 & -24 \\ 1 & -h & -30 \end{bmatrix} \begin{array}{l} R2 \pm (-5)R1 \\ R3 \pm -R1 \end{array} \begin{bmatrix} 1 & 6 & 40 \\ 0 & -32 & -224 \\ 0 & h-6 & -70 \end{bmatrix}$$

$$\begin{array}{l} R2 \cdot \frac{-1}{32} \\ \rightarrow \\ R3 \cdot (-1) \end{array} \begin{bmatrix} 1 & 6 & 40 \\ 0 & 1 & 7 \\ 0 & h+6 & 70 \end{bmatrix} \xrightarrow{R3 \pm -(h+6)R2} \begin{bmatrix} 1 & 6 & 40 \\ 0 & 1 & 7 \\ 0 & 0 & 70-(h+6)7 \end{bmatrix}$$

We need $70 - (h+6)7 = 0$

$70 = 7(h+6)$

$10 = h+6$

$\boxed{h=4}$

Alt Soln: Consider the system represented by the last two rows (i.e. removing the first eqn):

$$\begin{bmatrix} 5 & -2 & -24 \\ 3 & 2 & 8 \end{bmatrix} \xrightarrow{R2 \cdot 2} \begin{bmatrix} 5 & -2 & -24 \\ 6 & 4 & 16 \end{bmatrix} \xrightarrow{R2 \pm (-1)R1} \begin{bmatrix} 5 & -2 & -24 \\ 1 & 6 & 40 \end{bmatrix} \xrightarrow{R1 \leftrightarrow R2} \begin{bmatrix} 1 & 6 & 40 \\ 5 & -2 & -24 \end{bmatrix}$$

$$\xrightarrow{R2 \pm (-5)R1} \begin{bmatrix} 1 & 6 & 40 \\ 0 & -32 & -224 \end{bmatrix} \xrightarrow{R3 \cdot \frac{1}{-32}} \begin{bmatrix} 1 & 6 & 40 \\ 0 & 1 & 7 \end{bmatrix} \xrightarrow{R1 \pm (-6)R2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 7 \end{bmatrix}.$$

This system has a unique soln $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$. For the original system to be consistent, this must also be a soln to the first equation $(-1)x_1 + h x_2 = 30$. That is, we need

$(-1)(-2) + h(7) = 30$

$7h = 28$

$\boxed{h=4}$