Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. [1.4.14] Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or why not?

2. [1.4.{15,16}] For the matrices A and vectors $\mathbf{b}$ below, show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible $\mathbf{b}$, and describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ does have a solution.

   (a) $A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

   (b) $A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

3. [1.4.42] Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m?

4. [1.5.{7,11}] Given a matrix A and a vector $\mathbf{b}$, describe all solutions to $A\mathbf{x} = \mathbf{b}$ in parametric form.

   (a) $A = \begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$, $\mathbf{b} = \mathbf{0}$

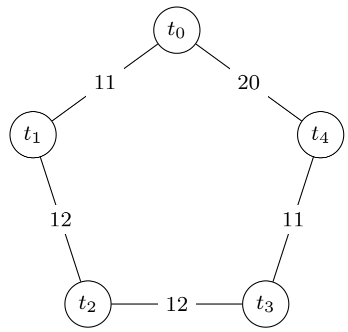
   (b) $A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, $\mathbf{b} = \mathbf{0}$

   (c) $A = \begin{bmatrix} 1 & 0 & 3 & 1 & -1 \\ 2 & -1 & 2 & 0 & 1 \\ -1 & 1 & 3 & 3 & 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$

5. Let A be an $(m \times n)$ matrix, let $\mathbf{v} \in \mathbb{R}^n$, and let c be a scalar. Prove that $A(c\mathbf{v}) = c(A\mathbf{v})$.

6. A game show has n treasure chests arranged in a circle, indexed from 0 through $n-1$. For $0 \leq i \leq n-1$, let t_i be the number of dollars in the ith chest; these amounts are not known to the contestant. On the line between the ith and $(i+1)$st treasure chest, there is a sign visible to the contestant that reveals the sum $t_i + t_{i+1}$ of the amounts in the ith and $(i+1)$st chests. The object of the game is for the contestant to select the chest with maximum value.

   (a) In this situation, which chest should the contestant pick? Justify your answer.

   (b) Give an example of a situation with four treasure chests where it is impossible for the contestant to know which chest to pick.

   (c) Characterize the values of n such that the contestant can determine the amounts t_i in the n chests from the given sums $t_i + t_{i+1}$, no matter what the sums may be.