Directions: You may work to solve these problems in groups, but all written work must be your own. Show all work; no credit for solutions without work..

1. [1.3.{11,12}] Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a_1}$, $\mathbf{a_2}$, and $\mathbf{a_3}$.

   (a) $\mathbf{a_1} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{a_2} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{a_3} = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$.

   (b) $\mathbf{a_1} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\mathbf{a_2} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$, $\mathbf{a_3} = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$.

2. [1.3.13] Determine if $\mathbf{b}$ is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

3. [1.3] True/False. Justify your answers.

   (a) Another notation for the vector $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$ is $[-4\ 3]$.

   (b) The points in the plane corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ lie on a line through the origin.

   (c) An example of a linear combination of vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ is the vector $\frac{1}{2}\mathbf{v_1}$.

   (d) The weights $c_1, \ldots, c_p$ in a linear combination $c_1\mathbf{v_1} + \cdots + c_p\mathbf{v_p}$ cannot all be zero.

   (e) The set $\mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.

4. Pivot columns in a matrix.

   (a) Let $A = [\ \mathbf{a_1}\ \mathbf{a_2}\ \cdots\ \mathbf{a_p}\ ]$ and let $B = [\ \mathbf{b_1}\ \mathbf{b_2}\ \cdots\ \mathbf{b_p}\ ]$. Suppose that $\mathbf{a_i} = \mathbf{a_j}$ and A is row equivalent to B. Explain why it must be that $\mathbf{b_i} = \mathbf{b_j}$. (Hint: how do the ith and jth columns behave with respect to each elementary row operation?)

   (b) Let $A = [\ \mathbf{a_1}\ \mathbf{a_2}\ \cdots\ \mathbf{a_p}\ ]$. Suppose that $\mathbf{a_i} = \mathbf{a_j}$ and that $i < j$. Use part (a) to explain why the jth column of A cannot be a pivot column. (Hint: let B be the reduced echelon form of A.)

5. [1.4.9] Write the system first as a vector equation and then as a matrix equation.

$$\begin{aligned} 3x_1 + x_2 - 5x_3 &= 9 \\ x_2 + 4x_3 &= 0 \end{aligned}$$

6. [1.4.11] Given A and $\mathbf{b}$ below, write augmented matrix corresponding to the matrix equation $A\mathbf{x} = \mathbf{b}$ and solve for $\mathbf{x}$.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$$