Directions:

1. Section: Math251
2. Write your name with one character in each box below.
3. Show all work. No credit for answers without work.
4. This assessment is closed book and closed notes. You may not use electronic devices, including calculators, laptops, and cell phones.

Academic Integrity Statement: I will complete this work on my own without assistance, knowing or otherwise, from anyone or anything other than the instructor. I will not use any electronic equipment or notes (except as permitted by an existing official, WVU-authorized accommodation).

Signature: _____

1. Let $f(x, y) = x + y^2$ and let $R = [1, 5] \times [2, 3]$.

   (a) **[8 points]** Evaluate $\iint_R f(x, y)\, dA$.

   (b) **[2 points]** What is the average value of $f(x, y)$ over R?

2. **[10 points]** Find the volume of the solid that lies under the hyperbolic paraboloid $z = 3y^2 - x^2 + 2$ and above the rectangle $R = [-1, 1] \times [1, 2]$.

3. **[10 points]** Evaluate $\iint_D xy\, dA$ where D is the triangular region with vertices $(0, 0)$, $(2, 2)$, and $(4, 0)$.

4. Let a, b, c, d be real numbers such that $0 < a < b$ and $0 < c < d < 2\pi$, and let R be the polar rectangle $\{(r, \theta) \colon\ a \leq r \leq b \text{ and } c \leq \theta \leq d\}$.

   (a) **[4 points]** Sketch R in the plane, using typical example values of a, b, c, d and labeling the appropriate features of your figure with these constants.

   (b) **[4 points]** Let $\Delta r = b - a$ and $\Delta\theta = d - c$. Without using polar integration, prove directly that the area of R equals $\frac{1}{2}(a + b)\Delta r \Delta\theta$.

   (c) **[2 points]** What formula to we obtain when Δr and $\Delta\theta$ tend to zero?

5. **[12 points]** Evaluate $\iint_R x^2 + y^2 \, dA$ where R is the region in the first quadrant of the circle with radius 2 centered at the origin.

6. **[12 points]** Evaluate $\displaystyle\int_0^1 \int_x^{2x} \int_{e^x}^{e^y} \frac{y+x}{z}\, dz\, dy\, dx.$

7. **[12 points]** Evaluate $\displaystyle\iiint_E z\, dV$, where E is the solid bounded inside the cylinder $x^2 + y^2 = 4$, below the cone $z = \sqrt{x^2 + y^2}$, and above the plane $z = 0$.

8. **[12 points]** Evaluate $\iiint_B (x^2 + y^2)\, dV$ where B is the ball of radius 3 centered at the origin.

9. **[12 points]** Find the volume of the part of the ball $\rho \leq a$ that lies between the cones $\phi = \pi/4$ and $\phi = \pi/3$.