Directions:

1. Section: Math251 007
2. Write your name with one character in each box below.
3. Show all work. No credit for answers without work.
4. This assessment is closed book and closed notes. You may not use electronic devices, including calculators, laptops, and cell phones.

Academic Integrity Statement: I will complete this work on my own without assistance, knowing or otherwise, from anyone or anything other than the instructor. I will not use any electronic equipment or notes (except as permitted by an existing official, WVU-authorized accommodation).

Signature: Solutions

1. Evaluate the following.

   (a) **[4 points]** $\int_{-1}^{3}\int_{0}^{3}\int_{0}^{2} xy^2z^3\,dz\,dy\,dx.$ $= \int_{-1}^{3}\int_{0}^{3} \frac{1}{4}xy^2z^4\Big|_{z=0}^{z=2} dy\,dx$

$$= \int_{-1}^{3}\int_{0}^{3} \frac{1}{4}xy^2\cdot 2^4 - 0\,dy\,dx = \int_{-1}^{3}\int_{0}^{3} 4xy^2\,dy\,dx = \int_{-1}^{3} \frac{4}{3}xy^3\Big|_{y=0}^{y=3} dx$$

$$= \int_{-1}^{3} \frac{4}{3}x(3)^3 - 0\,dx = \int_{-1}^{3} 4\cdot 3^2 x\,dx = 36\int_{-1}^{3} x\,dx = 36\left[\frac{1}{2}x^2\right]_{x=-1}^{x=3}$$

$$= 36\left[\frac{1}{2}3^2 - \frac{1}{2}(-1)^2\right] = 18\left[9-1\right] = \underset{2\cdot 3^2}{18}\cdot\underset{2^3}{8} = \boxed{144}$$

   (b) **[3 points]** $\int_{0}^{1}\int_{0}^{x}\int_{0}^{1} ye^{xz}\,dz\,dy\,dx.$ $= \int_0^1\int_0^x \left[\frac{y}{x}e^{xz}\right]_{z=0}^{z=1} dy\,dx$

$$= \int_0^1\int_0^x \frac{y}{x}e^x - \frac{y}{x}\cancel{e^0}^{\,1}\,dy\,dx = \int_0^1\int_0^x \frac{e^x-1}{x}\,y\,dy\,dx = \int_0^1 \left[\frac{e^x-1}{x}\cdot\frac{1}{2}y^2\right]_{y=0}^{y=x} dx$$

$$= \int_0^1 \frac{e^x-1}{x}\cdot\frac{1}{2}x^2 - 0\,dx = \frac{1}{2}\int_0^1 x(e^x-1)\,dx$$

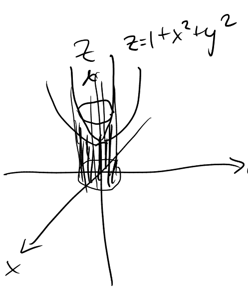
$u = x$ $\quad dv = e^x - 1$

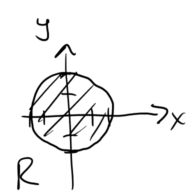
$du = dx$ $\quad v = e^x - x$

$$= \frac{1}{2}\left[x(e^x-x) - \int (e^x - x)\,dx\right]\Big|_0^1 = \frac{1}{2}\left[xe^x - x^2 - \left(e^x - \frac{x^2}{2}\right)\right]\Big|_0^1 = \frac{1}{2}\left[xe^x - e^x - \frac{1}{2}x^2\right]_0^1 = \frac{1}{2}\left(e^1 - e^1 - \frac{1}{2} - (0-1-0)\right)$$

$$= \frac{1}{2}\left(-\frac{1}{2}+1\right) = \boxed{\frac{1}{4}}$$

   (c) **[3 points]** Evaluate $\iiint_E e^z\,dV$ where E is enclosed by the paraboloid $z = 1 + x^2 + y^2$, the cylinder $x^2 + y^2 = 4$, and the xy-plane.





$$\iiint_E e^z\,dV = \iint_R \int_0^{1+x^2+y^2} e^z\,dz\,dA$$

$$= \int_0^{2\pi}\int_0^2\int_0^{1+r^2} e^z\,dz\;r\,dr\,d\theta = \int_0^{2\pi}\int_0^2 e^z\Big|_{z=0}^{z=1+r^2} r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 r\left(e^{1+r^2} - e^0\right)dr\,d\theta = \int_0^{2\pi}\int_0^2 re^{1+r^2} - r\,dr\,d\theta = \left(\int_0^{2\pi} d\theta\right)\left(\int_0^2 re^{1+r^2} - r\,dr\right)$$

$$= (2\pi)\left(\int_0^2 re^{1+r^2}\,dr - \int_0^2 r\,dr\right) = 2\pi\left(\frac{1}{2}\int_0^2 e^{1+r^2}\,2r\,dr - \left(\frac{1}{2}r^2\right)\Big|_{r=0}^{r=2}\right)$$

$u = 1+r^2$

$du = 2r\,dr$

$$= 2\pi\left(\frac{1}{2}\int_1^5 e^u\,du - \left(\frac{1}{2}\cdot 4 - 0\right)\right) = 2\pi\left(\frac{1}{2}e^u\Big|_{u=1}^{u=5} - 2\right) = 2\pi\left(\frac{1}{2}(e^5 - e) - 2\right)$$

$$= \boxed{\pi(e^5 - e - 4)}$$