

Directions:

1. Section: Math251 007
2. Write your name with one character in each box below.
3. Show all work. No credit for answers without work.
4. This assessment is closed book and closed notes. You may not use electronic devices, including calculators, laptops, and cell phones.

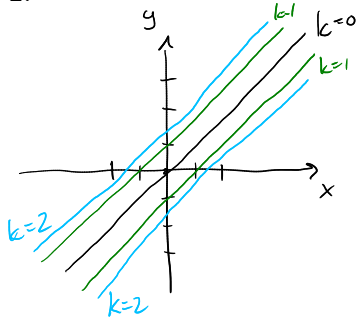
Academic Integrity Statement: I will complete this work on my own without assistance, knowing or otherwise, from anyone or anything other than the instructor. I will not use any electronic equipment or notes (except as permitted by an existing official, WVU-authorized accommodation).

Signature: Sduhns

1. Let $f(x, y) = (y - x)^2$.

(a) [2 points] Draw a contour map of f showing level curves for levels 0, 1, and 2.

$$\begin{aligned} f(x, y) &= k \\ (y - x)^2 &= k \\ y - x &= \pm \sqrt{k} \\ y &= x \pm \sqrt{k} \end{aligned}$$



(b) [2 points] Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

$$\begin{aligned} \frac{\partial f}{\partial x} &= 2(y - x) \cdot (-1) \\ &= 2(x - y) \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial y} &= 2(y - x) \cdot 1 \\ &= 2(y - x) \end{aligned}$$

2. [3 points] Use the linear approximation for $g(x, y) = e^{xy^2-2}$ at $(2, 1)$ to estimate $g(2.1, 0.8)$.

$$\begin{aligned} g_x &= e^{xy^2-2} \frac{\partial}{\partial x} [xy^2-2] \\ &= e^{xy^2-2} y^2 \\ g_x(2, 1) &= e^{2(1)^2-2} 1^2 \\ &= e^0 \cdot 1 = 1 \end{aligned}$$

$$\begin{aligned} g_y &= e^{xy^2-2} \frac{\partial}{\partial y} [xy^2-2] \\ &= e^{xy^2-2} \cdot 2xy \\ g_y(2, 1) &= e^{2 \cdot 1^2-2} \cdot 2 \cdot 2 \cdot 1 \\ &= 4 \end{aligned}$$

$$\begin{aligned} L(x, y) &= g(2, 1) + g_x(2, 1) \Delta x + g_y(2, 1) \Delta y \\ &= e^{2 \cdot 1^2-2} + 1(x - x_0) + 4(y - y_0) \\ L(2.1, 0.8) &= 1 + 1(0.1) + 4(-0.2) \\ &= 1 + 0.1 - 0.8 = \boxed{0.3} \end{aligned}$$

3. [3 points] An cylindrical container with tall sides has a circular base with adjustable radius. A fixed amount of liquid is poured into the cylinder. The radius of the cylinder increases at a rate of 2 centimeters per minute. Find the rate of change in the liquid's height when the cylinder has a radius of 10 centimeters and the liquid's height is 7 cm.

$$V = \pi r^2 h, \quad r \text{ and } h \text{ vary with time } t$$

Chain Rule:

$$\frac{dV}{dt} = \frac{\partial V}{\partial r} \cdot \frac{dr}{dt} + \frac{\partial V}{\partial h} \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\pi r h \cdot \frac{dr}{dt} + \pi r^2 \cdot \frac{dh}{dt}$$

Constant liquid volume means $\frac{dV}{dt} = 0$; $\frac{dr}{dt} = 2$,

$$0 = 2\pi r h \cdot 2 + \pi r^2 \frac{dh}{dt}$$

$$\pi r^2 \frac{dh}{dt} = -4\pi r h$$

$$\frac{dh}{dt} = \frac{-4\cancel{\pi}hr}{\cancel{\pi}r^2} = \frac{-4h}{r} = \frac{-4 \cdot 7}{10}$$

$$= \frac{-28}{10} = \boxed{-2.8 \text{ cm/min}}$$