Directions: Solve 5 of the following 6 problems. All written work must be your own, using only permitted sources. See the "General Guidelines and Advice" on the homework page for more details.

1. Determine the smallest m such that every 2-connected graph with n vertices has a 2-connected spanning subgraph with at most m edges.

2. Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.

3. Let G be a graph not containing a 4-cycle. Prove that $\chi(G) \leq \alpha'(G) + 2$.

4. Let t be a nonnegative integer. For each n with $n \geq 5t$, construct an n-vertex graph with chromatic number $n - 2t$ and clique number $n - 3t$.

5. Let G be a graph with no induced copy of the claw ($K_{1,3}$).

   (a) Show that in a proper coloring, each subgraph of G induced by the union of two color classes consists of paths and even cycles.

   (b) An *equitable* coloring of a graph is a proper vertex-coloring in which every pair of color classes differs in size by at most 1. Prove that G has an equitable coloring that is optimal (i.e. uses just $\chi(G)$ colors).

6. Let G be a graph with no induced copy of P_4, let $k = \omega(G)$, and let σ be an ordering of $V(G)$. Prove that with respect to σ, the greedy algorithm produces a proper k-coloring of G. Hint: show that if a vertex u receives color j, then u completes a j-clique with vertices of colors $\{1, 2, \ldots, j\}$ that precede u in σ.