

Directions: Solve 5 of the following 6 problems. All written work must be your own, using only permitted sources. See the “General Guidelines and Advice” on the homework page for more details.

1. Decomposition into disconnected subgraphs.
 - (a) Prove that the complement of a disconnected graph is connected.
 - (b) For $n \geq 3$, determine the smallest integer t such that K_n decomposes into t disconnected subgraphs.
2. Find a P_6 -decomposition of the Petersen graph or show that no such decomposition exists.
3. Let G be the 6-cycle plus an edge joining a pair of vertices at distance 3. Find a G -decomposition of K_7 or show that no such decomposition exists.
4. Let G be a graph with girth 4 in which every vertex has degree k . Prove that G has at least $2k$ vertices. Determine all such graphs with exactly $2k$ vertices.
5. Prove that a self-complementary graph with n vertices exists if and only if $n = 4k$ or $n = 4k+1$ for some integer k . Hint: When n is divisible by 4, generalize the structure of P_4 by splitting the vertices into four groups. For n of the form $n = 4k + 1$, add one vertex to the graph constructed for $n = 4k$.
6. Let G be a graph in which every vertex has degree 3. Prove that G decomposes into claws if and only if G is bipartite.